Analysis and Improvement of Steganography Protocol Based on Bell States in Noise Environment

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Abstract: In the field of quantum communication, quantum steganography is an important branch of quantum information hiding. In a realistic quantum communication system, quantum noises are unavoidable and will seriously impact the safety and reliability of the quantum steganographic system. Therefore, it is very important to analyze the influence of noise on the quantum steganography protocol and how to reduce the effect of noise. This paper takes the quantum steganography protocol proposed in 2010 as an example to analyze the effects of noises on information qubits and secret message qubits in the four primary quantum noise environments. The results show that when the noise factor of one quantum channel noise is known, the size of the noise factor of the other quantum channel can be adjusted accordingly, such as artificially applying noise, so that the influence of noises on the protocol is minimized. In addition, this paper also proposes a method of improving the efficiency of the steganographic protocol in a noisy environment.

1 Introduction

Quantum information hiding is a combination of classical information hiding and quantum cryptography and quantum communication technology. In the information age, information security involves all aspects of people's daily life. With the rapid development of network research [Qu, Keeney and Robitzsch (2016)], information security has become crucial and has triggered a lot of research on it [Pradeep, Mridula and Mohana (2016)]. As an important branch of information security, information hiding also plays an indispensable role. Quantum information hiding is based on the design philosophy of classical information hiding, and secret communication between communicators is realized by establishing a covert quantum channel in quantum cryptography or quantum communication technology. In 2001, Terhal et al. [Terhal, Divincenzo and Leung (2000)] proposed the first quantum information hiding protocol.

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Subsequently, quantum information hiding has developed rapidly.

Quantum steganography is an important sub-discipline of quantum information hiding. It hides secret information by embedding secret information in an ordinary quantum carrier. In recent years, a variety of quantum steganographic protocols have emerged in which different carrier media is used, such as Zhou et al. [Zhou, Qiu, Li et al. (2018)], image [Meng, Rice, Wang et al. (2018); Cao, Zhou, Sun et al. (2018); Qu, Zheng, Luo et al. (2017)], video [Nie, Xu, Feng et al. (2018); Qu, Chen and Ji (2017)], audio [Shamneed and Maya (2014)], etc. In 2002, Geabanacloche [Geabanacloche (2002)] used quantum error-correcting codes to hide secret information. In this protocol, secret information can also be used as watermark to authenticate the security and integrality of data. In 2003, Guo et al. [Guo and Guo (2012)] proposed a quantum-hidden protocol that based on the Bell state's preparation of uncertainty characteristics. In 2007, Matin [Martin (2007)] proposed a novel quantum steganography protocol based on the BB84 protocol [Bennett (1984)], analyzed the transparency and security of the protocol, and calculated the protocol's capacity in detail. In 2010, Shaw et al. [Shaw and Brun (2010, 2011)] proposed a quantum steganography protocol by sharing random quantum (or classical) keys. In 2012, Mihara [Mihara (2012)] proposed a quantum steganography protocol that embeds secret information in the phase of a quantum state in the process of quantum Fourier transform. Wei et al. [Wei, Chen, Niu et al. (2013, 2015)] proposed two quantum steganographic protocols based on quantum probability measurement. In 2015, Mihara [Mihara (2015)] proposed a quantum steganographic protocol that combines quantum error correction codes with pre-entanglement. In 2016, based on the novel enhanced quantum representation (NEQR), we proposed a quantum steganography algorithm [Qu, He and Ma (2016)] that embeds secret information in the second least significant bit and has better concealment.

Quantum noise is inevitable in the implementation of actual quantum systems, and it will seriously affect the security and reliability of quantum systems. In recent years, there have been more and more studies on the influence of quantum noise on quantum communication protocols [Qu, Chen and Ji (2017); Qu, Cheng, Luo et al. (2017); Qu, Wu, Wang et al. (2017); Qu, Chen, Ji et al. (2018)]. For example, Wang et al. [Wang and Qu (2016)] studied the effect of noise on the deterministic joint remote preparation of a single qubit state algorithm through GHZ channels. Guan et al. [Guan, Chen, Wang et al. (2014)] calculated the output state and fidelity of the JRSP algorithm under amplitude damping and phase damping noise and analyzed the effect of noise on the algorithm in detail. Ma et al. [Ma, Gao, Zhang et al. (2017)] proposed an algorithm for deterministic remotely preparing quantum states by Brown states and analyzed the effects of noise on the algorithm. Wang et al. [Wang, Qu, Wang et al. (2017)] studied the effect of noise on the deterministic joint remote preparation of arbitrary two-qubit state schemes by GHZ states. Although, there are many researches about the effects of quantum noise on quantum remote preparation schemes. We want to know what effect quantum noise will have on a quantum steganography protocol and how to counter the effect of noise on the protocol. Therefore, this paper uses the steganographic protocol [Qu, Chen, Zhou et al. (2010)] proposed in 2010 as an example to study the effects of the four main quantum noises on the protocol and how to improve the protocol in a noisy environment so that the protocol has a better performance in a noisy environment.

This article is organized as follows. In Section 2, we briefly review the steganographic protocol we proposed in 2010. In Section 3, four main quantum noise models are introduced, and the effect of noise on steganographic protocols is analyzed. In Section 4, the effects of noise channels on steganography are calculated under several possible realistic communication conditions. Finally, Section 5 summarizes the work of this paper and proposes a possible method to improve the efficiency of steganographic protocols in noisy environments.

2 Quantum steganography protocol

In the original quantum steganography protocol [Qu, Chen, Zhou et al. (2010)], the sender Alice wishes to send to the receiver Bob an information bits sequence in which two bits of secret information are hidden. The concrete steps of the quantum steganography protocol are as follows:

S1) Bob firstly prepares a large amount of $\left| \Psi^- \right\rangle_{AB}$ states, then sends each A particles to Alice through quantum channel. Alice obtains the particle set $S_A = [A_1, A_2, ..., A_n]$, and Bob possesses the particle set $S_B = [B_1, B_2, ..., B_n]$.

S2) After getting S_A, Alice randomly enter the control mode S3 or the information transmission mode S4.

S3) Control mode: Alice and Bob conduct eavesdropping detection. If there is an eavesdropping, Bob stops communicating with Alice. Otherwise, enter the information transmission mode S4.

S4) Information transmission mode: (a) According to the information bits sequence, Alice performs the corresponding unitary operations on S_A and then gets $S'_A = [A'_1, A'_2, ..., A'_n]$ and sends it back to Bob. After Bob receives S'_A, he can decode the information bits sequence by performing Bell measurements on S'_A and S'_B. (b) Alice selects two particles and enters secret information hiding mode.

S5) Secret message hiding mode: Alice selects the A'_m and A'_{m+1} from S'_A according to the secret information. The two Bell states formed by $A'_{m-1} B'_{m-1}$ and $A'_m B'_m$ must be consistent with the secret information, m can be sent to Bob through the classic channel using IBF or one-time pad. Later, Alice copies the information $A'_{m-1} B'_{m-1}$ carried to $A'_{m+1} B'_{m+1}$ by doing a same unitary operation U_{ij} on A'_{m+1}. Finally, Alice measures $A'_m A'_{m+1}$ under the Bell-basis for entanglement swapping and sends them back to Bob.

S6) Secret message decoding mode: After Bob obtains the value m, he performs Bell-basis measurements on $A'_m A'_{m+1}$ and $B'_m B'_{m+1}$, respectively. Bob can decode the hidden secret information through the secret information encoding rules.

3 The noisy quantum steganography protocol

In this section, we describe the four main quantum channel noises, as well as a model for analyzing the effect of noise on the steganography protocol.

3.1 Quantum channel noises

This section will introduce the four main noise environments in the quantum channel, i.e., amplitude-damping, phase-damping, bit-flip and depolarizing noise.

3.1.1 Amplitude damping

Amplitude damping noise describes the effect on the system caused by the loss of energy in the quantum system. The Kraus operator of amplitude damping noise is expressed as follows.

$$E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda} \end{pmatrix}, E_1 = \begin{pmatrix} 0 & \sqrt{\lambda} \\ 0 & 0 \end{pmatrix}, \tag{1}$$

where λ denotes the noise factor and satisfies $0 \le \lambda \le 1$.

3.1.2 Phase damping

Compared with amplitude damping, phase damping describes the loss of quantum information, which does not include loss of information due to energy loss. The Kraus operator of phase damping noise is described as follows.

$$E_0 = \sqrt{1-\lambda}I, E_1 = \sqrt{\lambda}\sigma_z, \tag{2}$$

where the noise factor satisfies the condition $0 \le \lambda \le 1$.

3.1.3 Bit flip

The bit-flip noise changes a qubit from $|0\rangle$ to $|1\rangle$ or from $|1\rangle$ to $|0\rangle$ with the probability λ. The Kraus operator of bit-flip noise is described as follows.

$$E_0 = \sqrt{1-\lambda}I, E_1 = \sqrt{\lambda}\sigma_x, \tag{3}$$

In which λ denotes the noise factor and satisfies $0 \le \lambda \le 1$.

3.1.4 Depolarizing

The depolarizing noise will replace one qubit with a completely maximal mixed state $I/2$ in the probability λ in the system. The Kraus operator is expressed as:

$$E_0 = \sqrt{1-\lambda}I, E_1 = \sqrt{\frac{\lambda}{3}}\sigma_x, E_2 = \sqrt{\frac{\lambda}{3}}\sigma_z, E_3 = \sqrt{\frac{\lambda}{3}}\sigma_y, \tag{4}$$

where $\sigma_x, \sigma_y, \sigma_z$ are standard Pauli matrixes and the noise factor satisfies $0 \le \lambda \le 1$.

3.2 The steganography protocol under quantum noises

For ease of analysis, we study the effect of noise on each Bell state with transmission information.

In Step S1, when Bob transmits particle A to Alice, particle A will be affected by quantum noise. The initial state of quantum can be written as

$$\rho_{ini} = \left|\Psi^-\right\rangle\left\langle\Psi^-\right|_{AB} = \frac{1}{2}\left(\left|01\right\rangle - \left|10\right\rangle\right)\left(\left\langle01\right| - \left\langle10\right|\right)_{AB}. \tag{5}$$

After Bob particles are transferred to Alice, the quantum state shared by Alice and Bob becomes

$$\rho_1 = \sum_i E_i^A\left(\lambda_{BC}\right)\rho_{ini}E_i^{A\dagger}\left(\lambda_{BC}\right), \tag{6}$$

in which E_i^A denotes the noise operator acting on qubit A, superscript denotes the transmitted particle.

Since this article mainly analyzes the influence of noise on the steganographic protocol, it is assumed that the eavesdropping detection has passed, skipping the Step S3: control mode and entering the Step S4: information transmission mode.

Alice performs corresponding unitary operations U_{ij} on A particles according to the information bits sequence, the quantum system becomes

$$\rho'_1 = U_{ij}^A \rho_1 U_{ij}^{A\dagger}. \tag{7}$$

Then, Alice sends A particle back to Bob through noisy quantum channel, the quantum system becomes

$$\rho_2 = \sum_j E_j^A(\lambda_{AC})\rho E_j^{A\dagger}(\lambda_{AC}). \tag{8}$$

In a noise-free environment, the quantum state Bob receives should be

$$\left|T\right\rangle = U_{ij}^A\left|\Psi^-\right\rangle_{AB}. \tag{9}$$

To describe the quantum noise effect on the quantum state, the fidelity is defined as follow.

$$F = \left\langle T \mid \rho_2 \left|T\right\rangle\right.. \tag{10}$$

The value F represents the degree of similarity between the two quantum states. The value of 1 indicates that the two quantum states are the same. The smaller the value F, the smaller the similarity of the two quantum states. Therefore, for the information bits sequence, the effect of noise can be measured by Eq. (10).

For secret messages, assume that the quantum noises are described as $E_i^{A_m}$ and $E_j^{A_{m+1}}$ when transmitting A_m and A_{m+1}, respectively. And the initial state of the quantum system that carries the secret messages is represented by

$$\rho_s = \left| \Psi^- \right\rangle_{A_m B_m} \otimes \left| \Psi^- \right\rangle_{A_{m+1} B_{m+1}} \left\langle \Psi^- \right|_{A_m B_m} \otimes \left\langle \Psi^- \right|_{A_{m+1} B_{m+1}} \tag{11}$$

After transmitting the particles A_m and A_{m+1}, the quantum system becomes

$$\rho_{s1} = \sum_{i1 \, j1} E_{i1}^{A_m} \left(\lambda_{BC1} \right) E_{j1}^{A_{m+1}} \left(\lambda_{BC2} \right) \left| \Psi^- \right\rangle_{A_m B_m} \otimes \left| \Psi^- \right\rangle_{A_{m+1} B_{m+1}} \left\langle \Psi^- \right|_{A_m B_m}$$

$$\otimes \left\langle \Psi^- \right|_{A_{m+1} B_{m+1}} E_{j1}^{A_{m+1} \, \dagger} \left(\lambda_{BC2} \right) E_{i1}^{A_m \, \dagger} \left(\lambda_{BC1} \right). \tag{12}$$

Alice performs an entanglement swapping on ρ_{s1}, i.e. a Bell-basis measurement on $A_m' A_{m+1}'$, m satisfies the consistency condition. After that, the quantum becomes ρ_{s1}', and then Alice sends $A_m A_{m+1}$ back to Bob through noisy quantum channel. Finally, Bob gets the quantum state as follow.

$$\rho_{s2} = \sum_{i_2 j_2} E_{i_2}^{A_m} \left(\lambda_{AC1} \right) E_{j_2}^{A_{m+1}} \left(\lambda_{AC2} \right) \rho_{s1}' E_{j_2}^{A_{m+1} \, \dagger} \left(\lambda_{AC2} \right) E_{i_2}^{A_m \, \dagger} \left(\lambda_{AC1} \right). \tag{13}$$

Bob performs Bell-basis measurements on $A_m' A_{m+1}'$ and $B_m' B_{m+1}'$, respectively. According to the measurement results, the secret message can be decoded.

4 The analyses of noise effects

This section will discuss the effects of noise on the quantum steganographic protocol in several possible scenarios. It should be pointed out that, in the steganography protocol, the carrier of the secret information is based on the carrier of the transmitted information. Therefore, we will analyze the impact of noise on the transmitted information, and then analyze the influence of noise on the secret information.

4.1 Effect on transmitted information

a) Considering Bob's quantum channel is in a noisy environment ($\lambda_{BC} \neq 0$) while Alice's quantum channel is protected from noise ($\lambda_{AC} = 0$). The fidelity for each type of noise can be written as

$$F_{AD,\varnothing} = \frac{1}{2} + \frac{1}{2} \sqrt{1 - \lambda_{BC}} - \frac{\lambda_{BC}}{4}, \tag{14}$$

$$F_{Phs,\varnothing} = 1 - \lambda_{BC}, \tag{15}$$

$$F_{BF,\varnothing} = 1 - \lambda_{BC}, \tag{16}$$

$$F_{D,\varnothing} = 1 - \lambda_{BC}. \tag{17}$$

The subscripts in the left of Eq. (14) to Eq. (17) represent the noise scenarios, *AD* denotes amplitude damping, *Phs* denotes phase damping, *BF* denotes bit flip and *D* denotes depolarizing. The fidelity in the scenario that Bob's quantum channel is affected by noise X ($X = \varnothing, AD, Phs, BF, D$), while Alice's quantum channel is not affected by
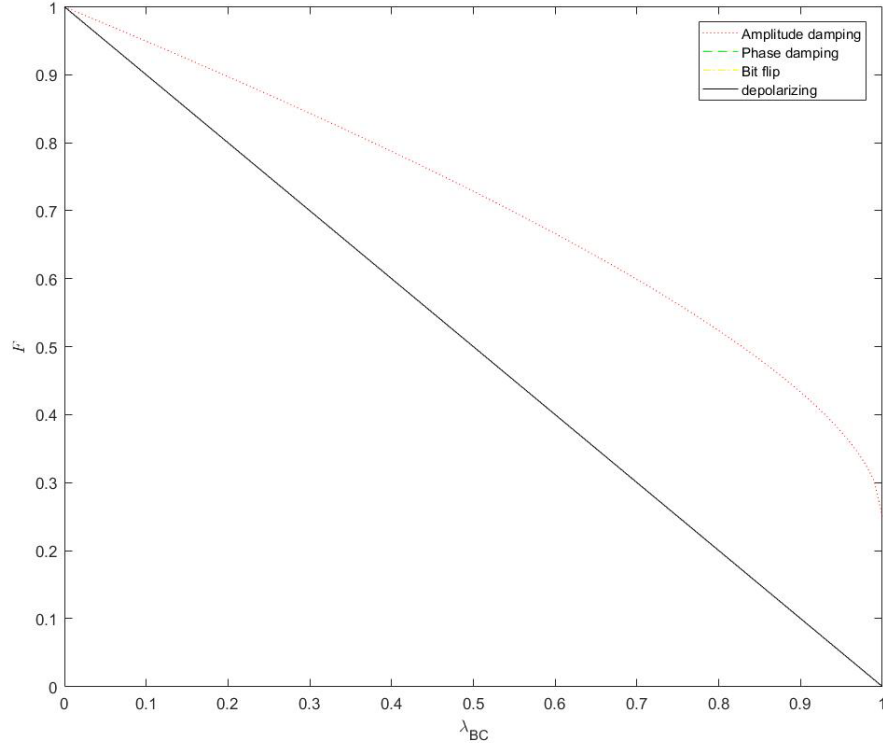
noise is described as $F_{X,\varnothing}$.

Figure 1: Fidelity of the transmitted information when only Bob's quantum channel is affected by a noisy environment

According to Fig. 1, we can see that the phase damping noise has less effect than the other three quantum noises whatever value λ_{BC} is. In addition, it is not difficult to find that the quantum state that carries information suffers from phase damping, bit flip and depolarizing noises have the same effect, and as the increase of λ_{BC}, the fidelity decreases linearly.

b) The quantum channel that Bob transmits particle and the quantum channel that Alice transmits particle are both affected by quantum noises ($\lambda_{BC} \neq 0$ and $\lambda_{AC} \neq 0$).

In the case that the quantum channel of Bob transmits particle is subjected to the amplitude damping noise and the quantum channel of Alice transmits particle is affected by one of the four different quantum noises, the fidelities in these four cases are calculated as follows.

$$F_{AD,AD} = \frac{1}{2} + \frac{1}{2}\sqrt{(1-\lambda_{BC})(1-\lambda_{AC})} - \frac{1}{4}(\lambda_{BC} + \lambda_{AC}) + \frac{1}{4}\lambda_{BC}\lambda_{AC} \qquad (18)$$

$$F_{AD,Phs} = \frac{1}{2} + \frac{1}{2}\left(1 - 2\lambda_{AC}\right)\sqrt{\left(1 - \lambda_{BC}\right)} - \frac{1}{4}\lambda_{BC} \tag{19}$$
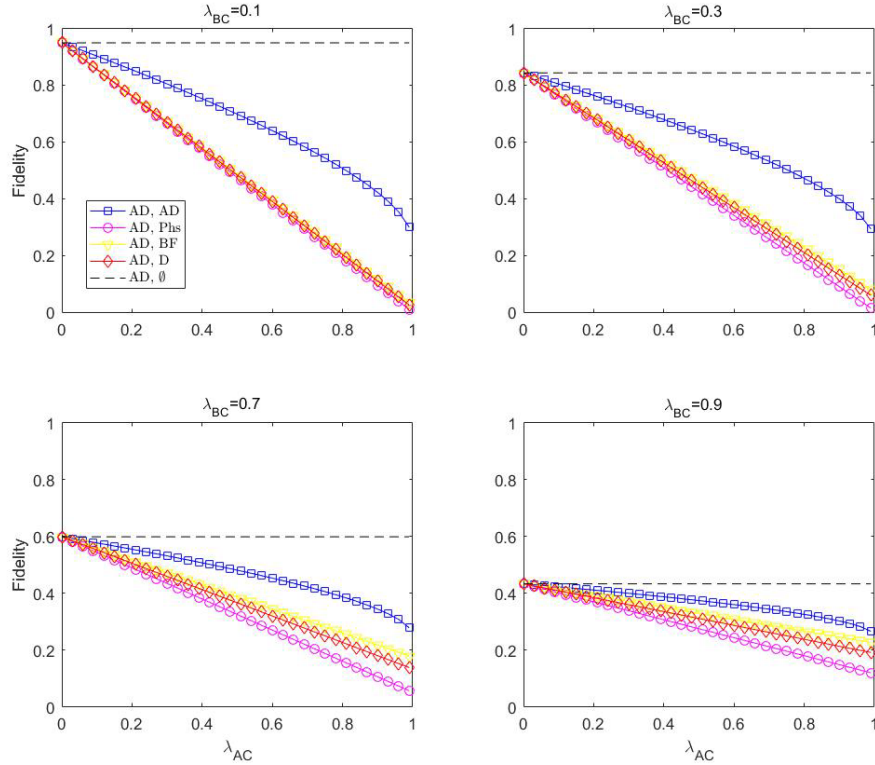

Figure 2: Fidelity of the transmitted information when Bob's quantum channel is affected by amplitude damping (AD) noise and Alice's quantum channel is affected by one of four noisy environments. The dash line denotes the situation only Bob's quantum channel is affected by amplitude damping (AD) noisy environment

$$F_{AD,BF} = \frac{1}{2} + \frac{1}{2}\left(1 - \lambda_{AC}\right)\sqrt{\left(1 - \lambda_{BC}\right)} - \frac{1}{2}\lambda_{AC} - \frac{1}{4}\lambda_{BC} + \frac{1}{2}\lambda_{BC}\lambda_{AC} \tag{20}$$

$$F_{AD,D} = \frac{1}{2} + \frac{1}{2}\left(1 - \frac{4}{3}\lambda_{AC}\right)\sqrt{\left(1 - \lambda_{BC}\right)} - \frac{1}{3}\lambda_{AC} - \frac{1}{4}\lambda_{BC} + \frac{1}{3}\lambda_{BC}\lambda_{AC} \tag{21}$$

In Fig. 2, we can see that when λ_{BC} is fixed, in other words, Bob's channel is affected by amplitude damping noise of the same intensity, the effect of noise of amplitude damping noise acted on Alice's channel is less than the other three quantum noises. When λ_{BC} is small ($\lambda_{BC} \leq 0.1$), phase damping, bit flip and depolarizing noise have a very close influence on transmission information. With the increase of λ_{BC}, the fidelity of bit flip noise is gradually higher than the depolarizing noise, and the depolarization noise is

gradually higher than the phase damping noise. In addition, as λ_{BC} increases, the fidelity at $\lambda_{AC} = 0$ gradually decreases, but the fidelity at $\lambda_{AC} = 1$ increases gradually. It can be found that when λ_{AC} is large, the fidelity increases as λ_{BC} increases. Which means that when one channel is affected by strong noise, the greater the noise that other channel is subjected to, the higher the fidelity of transmitted information is.
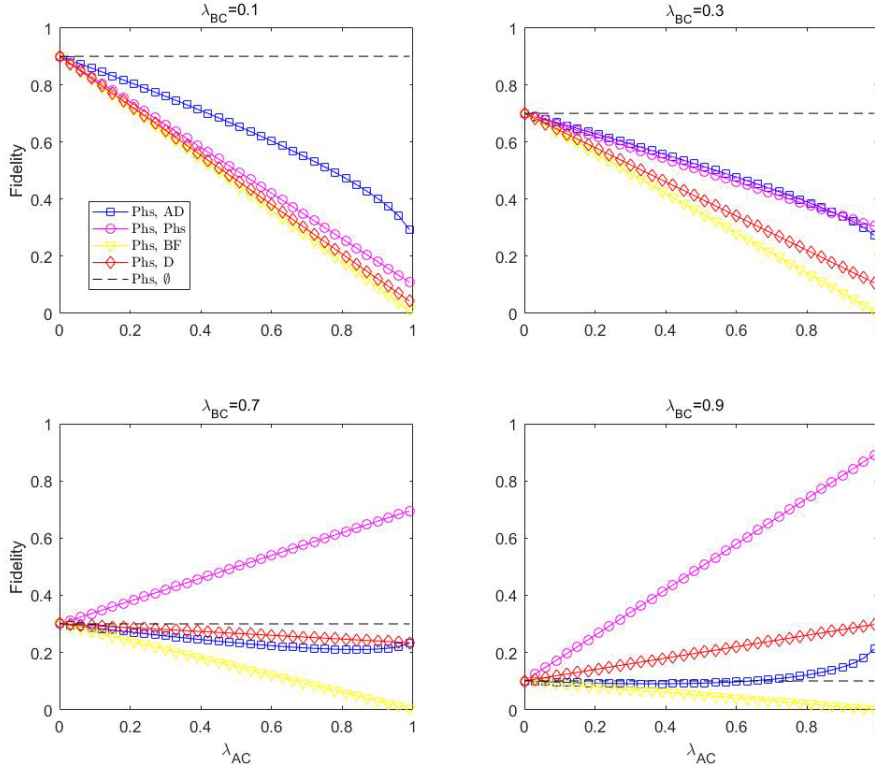

Figure 3: Fidelity of the transmitted information when Bob's quantum channel is affected by phase damping (*Phs*) noise and Alice's quantum channel is affected by one of four noisy environments. The dash line denotes the situation only Bob's quantum channel is affected by phase damping (*Phs*) noisy environment

In the case where the quantum channel of Bob transmits particle is subjected to the phase damping noise while the quantum channel of Alice transmits particle is affected by one of the four different quantum noises, the fidelities under the four cases are

$$F_{Phs,AD} = \frac{1}{2} + \frac{1}{2}\left(1 - 2\lambda_{BC}\right)\sqrt{1 - \lambda_{AC}} - \frac{\lambda_{AC}}{4} \tag{22}$$

$$F_{Phs,Phs} = 1 + 2\lambda_{BC}\lambda_{AC} - \lambda_{BC} - \lambda_{AC} \tag{23}$$

$$F_{Phs,BF} = 1 + \lambda_{BC}\lambda_{AC} - \lambda_{BC} - \lambda_{AC} \tag{24}$$

$$F_{Phs,D} = 1 + \frac{4}{3}\lambda_{BC}\lambda_{AC} - \lambda_{BC} - \lambda_{AC} \tag{25}$$

In Fig. 3, we plot Eq. (22) to Eq. (25) as a function of λ_{AC} for several values of λ_{BC}. It can be seen that when λ_{BC} is small ($\lambda_{BC} \leq 0.3$), the effect of phase damping acting on Alice's quantum channel is less than phase damping, phase damping is less than depolarizing noise, depolarizing noise is less than bit flip noise, and fidelity in the four noises decreases with increasing noise intensity. With the increase of λ_{BC}, the phase damping noise acting on Alice's channel is gradually smaller than the depolarizing noise, the depolarizing noise is smaller than the phase damping noise, and the phase damping noise is smaller than the bit flip noise. When λ_{BC} is large, the fidelity of transmitted information when Alice's channel subjected to phase damping or depolarizing noise increases with increasing λ_{AC}. Another interesting result is that, more phase damping noise or more depolarizing noise acting on Alice's channel can increase the fidelity of the transmitted information compared to the situation that Alice's channel is protected from noise when $\lambda_{BC} \geq 0.8$. Hence, if quantum noise is inevitable and Alice can choose different noisy communication channels, she can effectively improve the fidelity of transmitted information by selecting the right noise channel.

Next, we study the scene that Bob's quantum channel is subjected to the bit flip noise while the Alice's quantum channel is affected by one of the four different quantum noises. The fidelities are given as follows.

$$F_{BF,AD} = \frac{1}{2} + \frac{1}{2}\left(1 - \lambda_{BC}\right)\sqrt{1 - \lambda_{AC}} + \frac{1}{2}\lambda_{BC}\lambda_{AC} - \frac{1}{2}\lambda_{BC} - \frac{1}{4}\lambda_{AC} \tag{26}$$

$$F_{BF,Phs} = 1 + \lambda_{BC}\lambda_{AC} - \lambda_{BC} - \lambda_{AC} \tag{27}$$

$$F_{BF,BF} = 1 + 2\lambda_{BC}\lambda_{AC} - \lambda_{BC} - \lambda_{AC} \tag{28}$$

$$F_{BF,D} = 1 + \frac{4}{3}\lambda_{BC}\lambda_{AC} - \lambda_{BC} - \lambda_{AC} \tag{29}$$

In Fig. 4 we plot Eq. (26) to Eq. (29) as a function of λ_{AC} for several values of λ_{BC}. In this case, we can find similar conclusions as when Bob's channel is subjected to phase damping noise. When λ_{BC} is small, amplitude damping noise acting on Alice's channel has the least influence compared to other three noises. With the increase of λ_{BC}, bit flip noise acting on Alice's channel has the least influence on transmission information. When λ_{BC} is large, bit flipping, depolarizing noise and phase damping noise all increase with the increase of λ_{AC}. When Bob's quantum channel suffers from strong bit flip noise ($\lambda_{BC} > 0.75$), more bit flip noise, depolarizing noise, or phase damping noise acting on Alice's channel can increase the fidelity of the transmitted information.
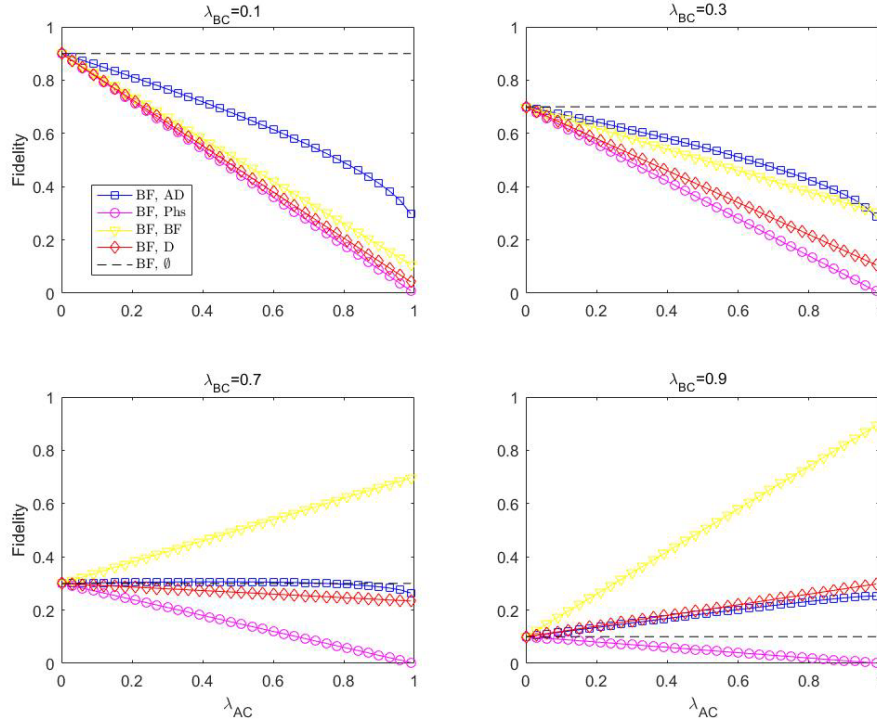
Figure 4: Fidelity of the transmitted information when Bob's quantum channel is affected by bit flip (*BF*) noise and Alice's quantum channel is affected by one of four noisy environments. The dash line denotes the situation only Bob's quantum channel is affected by bit flip (*BF*) noisy environment

Another important case is where the quantum channel of Bob transmits particle is subjected to the depolarizing noise while the quantum channel of Alice transmits particle is affected by one of the four different quantum noises, the fidelities under the four cases are calculated as follows.

$$F_{D,AD} = \frac{1}{2} + \frac{1}{2}\left(1 - \lambda_{BC}\right)\sqrt{\left(1 - \lambda_{AC}\right)} - \frac{1}{2}\lambda_{BC} - \frac{1}{4}\lambda_{AC} + \frac{5}{12}\lambda_{BC}\lambda_{AC} \tag{30}$$

$$F_{D,Phs} = 1 + \frac{4}{3}\lambda_{BC}\lambda_{AC} - \lambda_{BC} - \lambda_{AC} \tag{31}$$

$$F_{D,BF} = 1 + \frac{4}{3}\lambda_{BC}\lambda_{AC} - \lambda_{BC} - \lambda_{AC} \tag{32}$$

$$F_{D,D} = 1 + \frac{4}{3}\lambda_{BC}\lambda_{AC} - \lambda_{BC} - \lambda_{AC} \tag{33}$$

In Fig. 5 we plot Eq. (30) to Eq. (33) as a function of λ_{AC} for several values of λ_{BC}. It can be found that phase damping noise, bit flip noise, and depolarizing noise acting on

Alice's channel have the same effect on transmission information. And when λ_{BC} is small ($\lambda_{BC} \leq 0.7$), amplitude damping noise acting on Alice's channel has less effect on transmission information than the other three quantum noises. With the increase of λ_{BC}, the fidelity of the phase damping, bit flip and depolarizing noise acting on Alice's channel is gradually greater than the amplitude damping noise. When λ_{BC} is large, the fidelity of the transmitted information increases gradually with the increase of λ_{AC}.
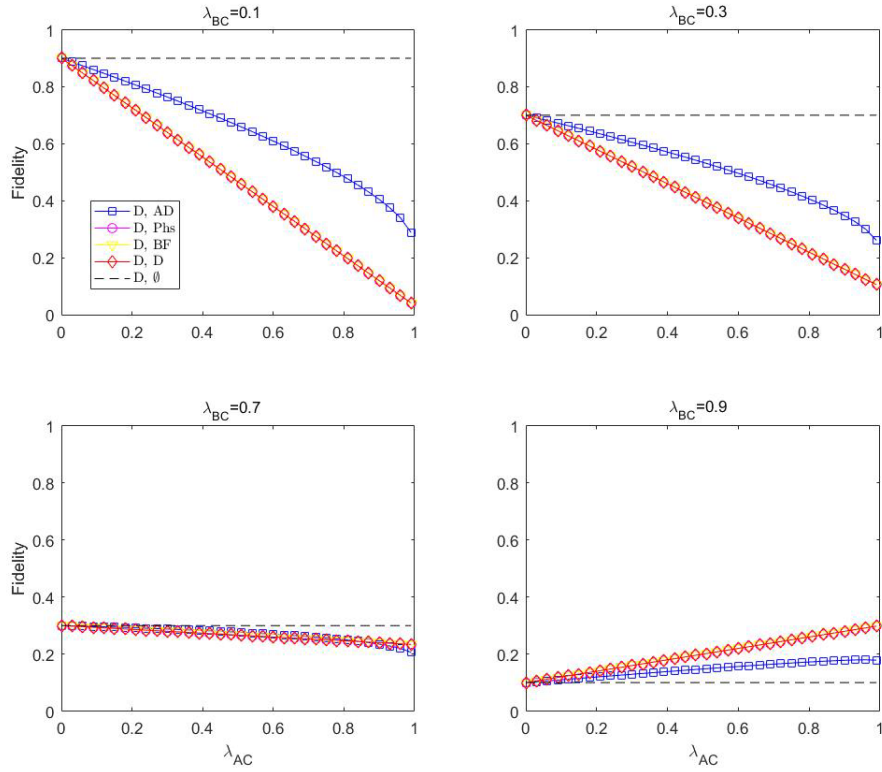

Figure 5: Fidelity of the transmitted information when Bob's quantum channel is affected by depolarizing (D) noise and Alice's quantum channel is affected by one of four noisy environments. The dash line denotes the situation only Bob's quantum channel is affected by depolarizing (D) noisy environment.

Analyzing Eq. (18) to Eq. (33), it can be found that $F_{AD,X} = F_{X,AD}$, $X = \varnothing, AD$, Phs, BF, $F_{Phs,X} = F_{X,Phs}$, $X = \varnothing, AD, Phs, BF, D$, $F_{BF,X} = F_{X,BF}$, $X = \varnothing, AD, Phs, BF, D$, $F_{D,X} = F_{X,D}$, $X = \varnothing, Phs, BF, D$. It shows that some noise has good symmetry. But such symmetry is not for $F_{AD,D}$ because $F_{AD,D} \neq F_{D,AD}$. In addition, $F_{Phs,Phs} = F_{BF,BF}$, which means that when two channels are simultaneously subjected to phase damping or bit flip, noise has the same effect on transmission information.
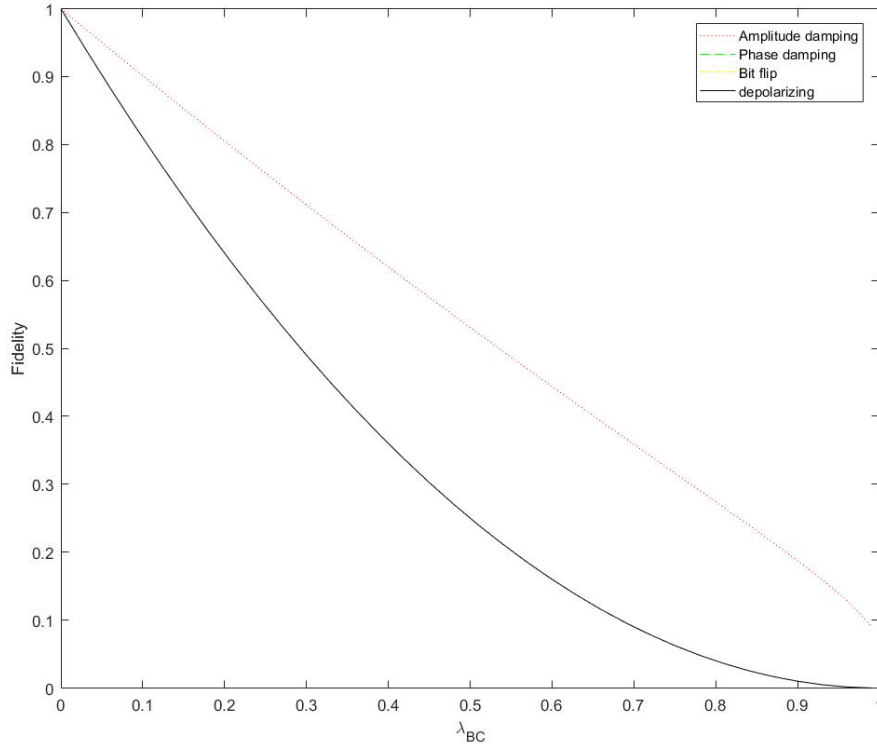
Figure 6: Efficiency of the secret message when only Bob's quantum channel is affected by a noisy environment

4.2 Effect on secret message

Taking the transmission of secret information 00 as an example, assume that in an ideal environment, the quantum system containing secret information is

$$\rho_{ini}^{00} = \left| \Psi^- \right\rangle_{A_m B_m} \otimes \left| \Psi^- \right\rangle_{A_{M+1} B_{M+1}} \left\langle \Psi^- \right|_{A_{M+1} B_{M+1}} \otimes \left\langle \Psi^- \right|_{A_m B_m} . \tag{34}$$

Considering that Bob sends his two qubits through a quantum channel simultaneously ($\lambda_{BC1} = \lambda_{BC2}$), and Alice sends her two qubits through a quantum channel at the same time ($\lambda_{AC1} = \lambda_{AC2}$). For convenient, the noise intensity of Bob's channel is expressed by λ_{BC}, and the noise intensity of Bob's channel is expressed by λ_{AC}.

Assuming Bob's quantum channel is affected by quantum noise while Alice's quantum channel is protected from quantum noise. In each type of noisy environment, the fidelity of secret information will be

$$F_{AD,\varnothing} = \frac{1}{2} - \frac{1}{2}\lambda_{BC} + \frac{1}{16}\lambda_{BC}^2 + \frac{1}{4}\sqrt{(1 - \lambda_{BC})}(2 - \lambda_{BC}) \tag{35}$$

$$F_{Phs,\varnothing} = 1 + \lambda_{BC}^2 - 2\lambda_{BC} \tag{36}$$

$$F_{BF,\varnothing} = 1 + \lambda_{BC}{}^2 - 2\lambda_{BC} \tag{37}$$

$$F_{D,\varnothing} = 1 + \lambda_{BC}{}^2 - 2\lambda_{BC} \tag{38}$$

In Fig. 6, we can see that the tendency of fidelity is similar to the fidelity of transmitted information (Fig. 1). The difference is that due to the superposition of noises of two transmitted information carriers, the middle part of the curve is concave, the fidelity of secret information is lower than transmitted information.

Next, we discuss the situation that both Alice and Bob's quantum channel is subjected to noise. If Bob's quantum channel is in amplitude damping noise and Alice's channel is affected by one of four quantum noises. The fidelities of secret information in these four cases are

$$F_{AD,AD} = \frac{1}{16} + \frac{1}{16}\left(1 - \lambda_{BC}\right)\left(1 - \lambda_{AC}\right)\left[6 + \left(1 - \lambda_{BC}\right)\left(1 - \lambda_{AC}\right)\right]$$
$$+ \frac{1}{4}\sqrt{\left(1 - \lambda_{BC}\right)\left(1 - \lambda_{AC}\right)}\left[1 + \left(1 - \lambda_{BC}\right)\left(1 - \lambda_{AC}\right)\right] \tag{39}$$

$$F_{AD,Phs} = \frac{1}{16}\left(1 + 2\lambda_{AC}{}^2 - 2\lambda_{AC}\right)\left(8 + \lambda_{BC}{}^2 - 8\lambda_{BC}\right) + \frac{1}{8}\lambda_{BC}{}^2\lambda_{AC}\left(1 - \lambda_{AC}\right)$$
$$+ \frac{1}{4}\left(1 - 2\lambda_{AC}\right)\left(2 - \lambda_{BC}\right)\sqrt{1 - \lambda_{BC}} \tag{40}$$

$$F_{AD,BF} = \frac{1}{16}\left(1 - \lambda_{AC}\right)^2\left(2 + 2\sqrt{1 - \lambda_{BC}} - \lambda_{BC}\right)^2 + \frac{1}{16}\lambda_{BC}{}^2\lambda_{AC}{}^2$$
$$+ \frac{1}{8}\lambda_{BC}\lambda_{AC}\left(1 - \lambda_{AC}\right)\left(2 + 2\sqrt{1 - \lambda_{BC}} - \lambda_{BC}\right) \tag{41}$$

$$F_{AD,D} = \frac{1}{16}\left(1 - \lambda_{AC}\right)^2\left(2 + 2\sqrt{1 - \lambda_{BC}} - \lambda_{BC}\right)^2 + \frac{1}{144}\lambda_{AC}{}^2\left(2 - 2\sqrt{1 - \lambda_{BC}}\right.$$
$$\left. - \lambda_{BC}\right)^2 + \frac{1}{24}\lambda_{BC}{}^2\lambda_{AC}\left(1 - \lambda_{AC}\right) \tag{42}$$

It can be seen, the effect of noise on a quantum system with secret message (Fig. 7) is similar to the quantum system with transfer information (Fig. 2). As to the transmission information, the first qubit in the Bell state is affected by noise. As to the secret message, it is based on the carrier of transmission information and only extended to two Bell states. And the entanglement swap does not change the effect of noise on the transmitted qubits.
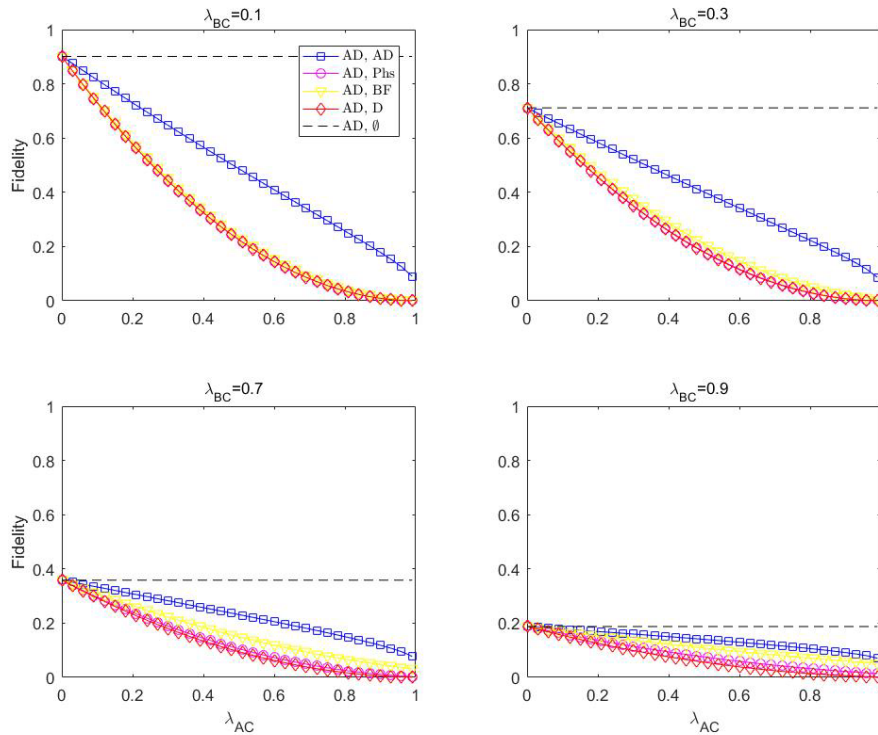
Figure 7: Efficiency of the secret message when Bob's quantum channel is affected by amplitude damping noise (*AD*) and Alice's quantum is affected by one of four quantum noises. Dash line denotes the fidelity of secret message when there is only Bob's quantum channel is affected by quantum noise

Therefore, in this case, the impact of noise on the secret message contains the same conclusion as the transmitted information.

5 Conclusions

In this paper, we investigate how the steganography protocol in Qu et al. [Qu, Chen, Zhou et al. (2010)] is affected by quantum noise and establish a noise impact model. After that, we analyzed the impact of noise on information bits and secret information bits, respectively. The results show that when the quantum channel of only one of Bob and Alice is affected by noise, amplitude-damping noise has the least impact on the protocol compared to the other three noises. When the quantum channels of both Bob and Alice are affected by noise, the fidelity of the quantum state that contains information can be improved by adjusting the size of noise factors of the two parties, thereby improving the efficiency of the protocol.

In order to resist the influence of noise, we consider that in Fortes et al. [Fortes and Rigolin (2015)], the author proposed that a qubit in the transmission is affected by quantum noise and another qubit is not affected by noise, if the second qubit is also affected by quantum noise, the efficiency of quantum teleportation will increase. This

means that the superposition of quantum noises may reduce the effect of quantum noise on the quantum state. Based on this idea, we can consider that the quantum channels used by Alice and Bob is suffered a noise that is a combination of different quantum noises to reduce the influence of noise on the steganographic protocol and improve the efficiency of the protocol in a noisy environment.

Therefore, the protocol can improve the efficiency of transmitting information by artificially applying additional quantum noise in a noisy environment. The invisibility and security of the noisy protocol is based on the invisibility and security of the original protocol, and therefore, it also has good invisibility and security.

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