CMC, vol.59, no.1, pp.215-227, 2019

An Asymmetric Controlled Bidirectional Quantum State Transmission Protocol

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Abstract: In this paper, we propose an asymmetric controlled bidirectional transmission protocol. In the protocol, by using the thirteen-qubit entangled state as the quantum channel, Alice can realize the transmission of a two-qubit equatorial state for Bob and Bob can transmit a four-qubit equatorial state for Alice under the control of Charlie. Firstly, we give the construction of the quantum channel, which can be done by performing several *H* and *CNOT* operations. Secondly, through implementing the appropriate measurements and the corresponding recovery operations, the desired states can be transmitted simultaneously, securely and deterministically. Finally, we analyze the performance of the protocol, including the efficiency, the necessary operations and the classical communication costs. And then, we describe some comparisons with other protocols. Since our protocol does not require auxiliary particles and additional operations, the classic communication costs less while achieving the multi-particle bidirectional transmission, so the overall performance of the protocol is better.

Keywords: Controlled, asymmetric, bidirectional quantum state transmission, remote state preparation.

1 Introduction

By using the quantum entanglement resources, a variety of quantum communication protocols were put forward to solve the problem of quantum information transmission, such as quantum secure direct communication [Wang, Deng, Li et al. (2005)], quantum secret sharing [Lin, Guo, Xu et al. (2016); Chen, Tang, Xu et al. (2018)], quantum key management [Xu, Chen, Duo et al. (2015); Liu, Xu, Yang et al. (2018)], quantum steganography [Wei, Chen, Niu et al. (2015); Qu, Cheng, Liu et al. (2018); Qu, Chen, Ji et al (2018)], quantum teleportation (QT) [Fortes and Rigolin (2017); Bennett, Brassard, Crépeau et al. (1993); Tan, Li and Yang (2018)] and remote state preparation (RSP) [Pati (2000); Cavaillès, Le Jeannic, Raskop et al. (2017]. In a RSP, the sender (Alice) can prepare a known state for a remote receiver (Bob) via a shared quantum channel and some classical communications. So far, many different protocols were proposed, such as

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CMC. doi: 10.32604/cmc.2019.05253

controlled RSP [Huang and Zhao (2017); Chen, Ma, Su et al. (2012); Liu and Hwang (2014); Wang, Zeng and Li (2015)], joint RSP [Choudhury and Dhara (2015); An (2010); Chen, Xia, Song et al. (2010)], deterministic RSP [Xu, Chen, Dou, et al. (2016); Chen, Su, Xu et al. (2014), Luo, Chen, Ma et al. (2010); Qu, Wu, Wang et al. (2017)] and low-entanglement RSP [Devetak and Berger (2001)]. In the CRSP protocol, some re-searchers also pay attention to the related research of controller's power [Li and Shohini (2017)] while introducing the controller. After that, some variants of quantum teleportation and RSP protocols were proposed: bidirectional teleportation [Kiktenko, Popov and Fedorov (2016); Li, Li, Sang et al. (2013)], hierarchical RSP [Shukla, Thapliyal and Pathak (2017)] and controlled bidirectional RSP (CBRSP) [Cao and Nguyen (2013); Sharma, Shukla, Banerjee et al. (2015); Peng, Bai and Mo (2015); Wang and Mo (2017); Zhang, Zha, Duan et al. (2016a, 2016b); Sang (2017); Song, Ni, Wang et al. (2017); Wu, Zha and Yang (2018); Chen, Sun, Xu et al. (2017); Sang and Nie (2017); Fang and Jiang (2018); Ma, Chen, Li et al. (2017)], for example. There are also some related quantum communication protocols that have been experimentally implemented [Zhang, Goebel, Wagenknecht et al. (2006); Sisodia, Shukla, Thapliyal et al. (2017); Rådmark, Wieśniak, Żukowski et al. (2013); Luo, Chen, Yang et al. (2012); Liu, Gao, Yu et al. (2018); Liu, Wang, Yuan et al. (2016)]. In this paper, we focus on the study of CBRSP.

Cao et al. [Cao and Nguyen (2013)] presented the first CBRSP protocol, which realizes the bidirectional transmission of the single-qubit state through some classical communications and local operations. After that, by using different entangled states as the quantum channel, many protocols [Sharma, Shukla, Banerjee et al. (2015); Peng, Bai and Mo (2015); Wang and Mo (2017); Zhang, Zha, Duan et al. (2016a, 2016b); Sang (2017); Song, Ni, Wang et al. (2017); Wu, Zha and Yang (2018)] can achieve the bidirectional transmission of single-qubit state. Sharma et al. [Sharma, Shukla, Banerjee et al. (2015)] gave three protocols, including the probabilistic, the deterministic and the joint CBRSP protocols. In Peng et al. [Peng, Bai and Mo (2015); Wang and Mo (2017)], two five-party joint CBRSP protocols were presented via the eight-and seven-qubit entangled state as the quantum channel, respectively. Furthermore, in these protocols [Cao and Nguyen (2013); Sharma, Shukla, Banerjee et al. (2015); Zhang, Zha, Duan et al. (2016a, 2016b); Sang (2017); Song, Ni, Wang et al. (2017)], the participants need the help of auxiliary particles and additional operations to complete the transmission task. Specifically, two controlled bidirectional hybrid of RSP and QT protocols were proposed by Sang [Sang (2017)] and Wu et al. [Wu, Zha and Yang (2018)], respectively. In 2017, by using thirteen-qubit entangled state as the quantum channel, a CBRSP protocol was presented by Chen et al. [Chen, Sun, Xu et al. (2017)], where Alice and Bob can transmit an arbitrary three-qubit state to each other simultaneously.

The above CBRSP protocols are symmetric. Moreover, there are many asymmetric protocols. Song et al. [Song, Ni, Wang et al. (2017)] put forward an asymmetric bidirectional RSP protocol to prepare single- and two-qubit state. An asymmetric bidirectional hybrid of RSP and QT protocol was proposed by Sang et al. [Sang and Nie (2017)], where Alice teleports a single-qubit state to Bob and Bob prepares a two-qubit state to Alice at the same time. Fang et al. [Fang and Jiang (2018)] investigated two asymmetric protocols for bidirectional hybrid of RSP and QT, where also studied bidirectional transmission of single- and two-qubit state. Further-more, in Ma et al. [Ma, Chen, Li et al. (2017)], the senders complete the bidirectional transmission of a four-qubit cluster-type state and a single-qubit state. However, in these protocols [Song, Ni, Wang et al. (2017); Fang and Jiang (2018); Ma, Chen, Li et al. (2017)], the sender or receiver require the auxiliary particles and additional operations. And the multi-qubit state prepared by protocol in Ma et al. [Ma, Chen, Li et al. (2017)] is only a special form. Chen et al. [Chen, Su, Xu et al. (2014)] investigated the quantum state secure transmission of equatorial and general state, respectively. Wei et al. [Wei, Shi, Zhu et al. (2018)] presented a protocol for remotely preparing an arbitrary n-qubit equatorial state via n two-qubit maximally entangled states as the quantum channel. Inspired by their protocols, we study the bidirectional transmission of two- and four-qubit equatorial state.

In this paper, we propose a controlled bidirectional transmission protocol. With the control of Charlie, Alice and Bob can realize the bidirectional transmission of two- and four-qubit equatorial state. First, by using several *H* and *CNOT* operations, the thirteen-qubit entangled state can be constructed as the quantum channel. Second, through carrying out the proper measurement and recovery operations, Alice and Bob can recover the prepared state simultaneously, securely and deterministically. In the end, we analyze the performance of the protocol and describe some comparisons with other protocols, including the efficiency, the necessary operations and the classical communication costs. After that, since our protocol does not require auxiliary particles and additional operations, the classic communication costs less while achieving the bidirectional transmission of two- and four-qubit equatorial state, so the protocol has the better overall performance.

The paper is structured as follows. We propose a controlled bidirectional transmission protocol in Section 2. Then, in Section 3, some discussions are given. In the end, we describe the conclusions in Section 4.

2 The controlled bidirectional quantum state transmission protocol

In this section, we describe the CBRSP protocol in detail. In the protocol, under the control of Charlie, Alice transmits a two-qubit equatorial state to Bob and Bob transmits a four-qubit equatorial state to Alice by using the thirteen-qubit entangled state as the quantum channel. We give the construction of the quantum channel at first. Then, the CBRSP protocol is described.

2.1 Quantum channel

The quantum channel is constructed according to the principles in Thapliyal et al. [Thapliyal, Verma and Pathak (2015)]. The thirteen-qubit product state $|\psi\rangle$ is used as an input state, where

$$|\psi\rangle=|0\rangle_1|0\rangle_2|0\rangle_3|0\rangle_4|0\rangle_5|0\rangle_6|0\rangle_7|0\rangle_8|0\rangle_9|0\rangle_a|0\rangle_b|0\rangle_c|0\rangle_d \ . \tag{1}$$

We construct the thirteen-qubit entangled state as the quantum channel by using *H* and *CNOT* operations as follows.

Firstly, we implement an *H* operation on particle 1. $|\psi\rangle$ is transformed to $|\psi_1\rangle$:

$$|\psi_1\rangle = \frac{(|0\rangle + |1\rangle)_1}{\sqrt{2}} |0\rangle_2 |0\rangle_3 |0\rangle_4 |0\rangle_5 |0\rangle_6 |0\rangle_7 |0\rangle_8 |0\rangle_9 |0\rangle_a |0\rangle_b |0\rangle_c |0\rangle_d \ . \tag{2}$$

Secondly, six *CNOT* operations are operated on the particle pairs (1, 2), (1, 4), (1, 6), (1, 8), (1, *a*), (1, *c*), where the particle 1 is used as the controlled particle and each of six particles 2, 4, 6, 8, *a*, *c* are used as the target particles. $|\psi_1\rangle$ is transformed to $|\psi_2\rangle$:

$$|\psi_2\rangle = \frac{1}{\sqrt{2}} (|0000000000000\rangle + |1111111111111\rangle)_{123456789abcd} \ . \tag{3}$$

Thirdly, after executing six *H* operations on the particles 2, 4, 6, 8, *a*, *c*, we carry out six *CNOT* operations on the particle pairs (2, 3), (4, 5), (6, 7), (8, 9), (*a*, *b*), (*c*, *d*), where the particles 2, 4, 6, 8, *a*, *c* are used as the controlled particles and 3, 5, 7, 9, *b*, *d* are used as the target particles, respectively.

Finally, we can construct the thirteen-qubit entangled state $|\psi\rangle'$ as the quantum channel:

$$\begin{aligned} |\psi\rangle' = \frac{1}{\sqrt{2}} (&|0\rangle_1 |\Phi^+\rangle_{23} |\Phi^+\rangle_{45} |\Phi^+\rangle_{67} |\Phi^+\rangle_{89} |\Phi^+\rangle_{ab} |\Phi^+\rangle_{cd} \\ &+ |1\rangle_1 |\Phi^-\rangle_{23} |\Phi^-\rangle_{45} |\Phi^-\rangle_{67} |\Phi^-\rangle_{89} |\Phi^-\rangle_{ab} |\Phi^-\rangle_{cd}), \end{aligned} \tag{4}$$

where $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$.

The generation and maintenance of the multi-qubit entangled state is a difficult task, but even so, some experiments results [Zhang, Goebel, Wagenknecht et al. (2006); Sisodia, Shukla, Thapliyal et al. (2017); Rådmark, Wieśniak, Żukowski, et al. (2013); Luo, Chen, Yang et al. (2012)] have been put forward to study the multi-qubit entangled state. As a consequence, by using some advanced technology, this task can be completed no longer difficult. Moreover, the bidirectional transmission task can be performed through the controlled bidirectional teleportation and preparation protocol. It is shown that the task can be performed with only Bell states [Thapliyal and Pathak (2015)]. In this paper, we mainly research the CBRSP. That is to say, the controller is needed in our protocol. Here, we use this thirteen-qubit entangled state as the quantum channel to accomplish the bidirectional preparation of two- and four-qubit states.

2.2 The controlled bidirectional quantum state transmission protocol

In this section, inspired by protocols [Chen, Su, Xu et al. (2014); Wei, Shi, Zhu et al. (2018)], we study the bidirectional transmission of two-and four-qubit equatorial state and will describe our CBRSP protocol in detail. The process of our protocol is graphically described in Fig. 1.

In the protocol, Alice can transmit two-qubit equatorial state $|\phi_{A2}\rangle$ to Bob and Bob can transmit four-qubit equatorial state $|\phi_{B4}\rangle$ to Alice, where

$$|\phi_{A2}\rangle = \frac{1}{2}\left(|00\rangle + e^{i\theta_1}|01\rangle + e^{i\theta_2}|10\rangle + e^{i\theta_3}|11\rangle\right), \tag{5}$$

$$
\begin{aligned}
\left|\phi_{B4}\right\rangle = \frac{1}{4}(&\left|0000\right\rangle + e^{i\beta_1}\left|0001\right\rangle + e^{i\beta_2}\left|0010\right\rangle + e^{i\beta_3}\left|0011\right\rangle + e^{i\beta_4}\left|0100\right\rangle + e^{i\beta_5}\left|0101\right\rangle \\
&+ e^{i\beta_6}\left|0110\right\rangle + e^{i\beta_7}\left|0111\right\rangle + e^{i\beta_8}\left|1000\right\rangle + e^{i\beta_9}\left|1001\right\rangle + e^{i\beta_{10}}\left|1010\right\rangle \\
&+ e^{i\beta_{11}}\left|1011\right\rangle + e^{i\beta_{12}}\left|1100\right\rangle + e^{i\beta_{13}}\left|1101\right\rangle + e^{i\beta_{14}}\left|1110\right\rangle + e^{i\beta_{15}}\left|1111\right\rangle),
\end{aligned}
\tag{6}
$$

here θ_i, β_j are real, $0 \le \theta_i, \beta_j \le 2\pi$, $i = 1, 2, 3$, $j = 1, 2, ..., 15$.

[A-1] Three participants pre-share the thirteen-qubit entangled state $\left|\psi\right\rangle'$. Specifically, Charlie holds the particle 1. The particles 2, 4, 6, 8, *a*, *c* belong to Alice and the particles 3, 5, 7, 9, *b*, *d* belong to Bob.

[A-2] Alice makes two-qubit measurement on her own particles 2, 4. The measurement basis $\{\left|A_0\right\rangle, \left|A_1\right\rangle, \left|A_2\right\rangle, \left|A_3\right\rangle\}$ is

$$
\begin{pmatrix} \left|A_0\right\rangle \\ \left|A_1\right\rangle \\ \left|A_2\right\rangle \\ \left|A_3\right\rangle \end{pmatrix}^T = \frac{1}{2}\begin{pmatrix} 1 & e^{-i\theta_1} & e^{-i\theta_2} & e^{-i\theta_3} \\ 1 & ie^{-i\theta_1} & -e^{-i\theta_2} & -ie^{-i\theta_3} \\ 1 & -e^{-i\theta_1} & e^{-i\theta_2} & -e^{-i\theta_3} \\ 1 & -ie^{-i\theta_1} & -e^{-i\theta_2} & ie^{-i\theta_3} \end{pmatrix} \begin{pmatrix} \left|00\right\rangle \\ \left|01\right\rangle \\ \left|10\right\rangle \\ \left|11\right\rangle \end{pmatrix}^T ,
\tag{7}
$$

Bob implements four-qubit measurement on his own particles 7, 9, *b*, *d*. The measurement basis $\{\left|\mathrm{B}_h\right\rangle \mid h = 0, 1, 2, ..., 15\}$ is

$$
\begin{aligned}
\left|\mathrm{B}_h\right\rangle = \frac{1}{4}(&\left|0000\right\rangle + e^{\frac{h\pi i}{8} - i\beta_1}\left|0001\right\rangle + e^{\frac{2h\pi i}{8} - i\beta_2}\left|0010\right\rangle + e^{\frac{3h\pi i}{8} - i\beta_3}\left|0011\right\rangle + e^{\frac{4h\pi i}{8} - i\beta_4}\left|0100\right\rangle \\
&+ e^{\frac{5h\pi i}{8} - i\beta_5}\left|0101\right\rangle + e^{\frac{6h\pi i}{8} - i\beta_6}\left|0110\right\rangle + e^{\frac{7h\pi i}{8} - i\beta_7}\left|0111\right\rangle + e^{\frac{8h\pi i}{8} - i\beta_8}\left|1000\right\rangle \\
&+ e^{\frac{9h\pi i}{8} - i\beta_9}\left|1001\right\rangle + e^{\frac{10h\pi i}{8} - i\beta_{10}}\left|1010\right\rangle + e^{\frac{11h\pi i}{8} - i\beta_{11}}\left|1011\right\rangle + e^{\frac{12h\pi i}{8} - i\beta_{12}}\left|1100\right\rangle \\
&+ e^{\frac{13h\pi i}{8} - i\beta_{13}}\left|1101\right\rangle + e^{\frac{14h\pi i}{8} - i\beta_{14}}\left|1110\right\rangle + e^{\frac{15h\pi i}{8} - i\beta_{15}}\left|1111\right\rangle).
\end{aligned}
\tag{8}
$$

We can rewrite the quantum channel $\left|\psi\right\rangle'$ as:

$$
\begin{aligned}
\left|\psi\right\rangle' = \frac{1}{8\sqrt{2}}\{&\left|0\right\rangle_1 [\sum_{m=0}^{3}(\left|A_m\right\rangle_{2,4}\left|P_m\right\rangle_{3,5})\sum_{h=0}^{15}(\left|B_h\right\rangle_{7,9,b,d}\left|Q_h\right\rangle_{6,8,a,c})] \\
&+ \left|1\right\rangle_1 [\sum_{m=0}^{3}(\left|A_m\right\rangle_{2,4}\left|P'_m\right\rangle_{3,5})\sum_{h=0}^{15}(\left|B_h\right\rangle_{7,9,b,d}\left|Q'_h\right\rangle_{6,8,a,c})]\},
\end{aligned}
\tag{9}
$$

Where

$$\left|P_0\right\rangle = \frac{1}{2}(1, e^{i\theta_1}, e^{i\theta_2}, e^{i\theta_3})(\left|00\right\rangle, \left|01\right\rangle, \left|10\right\rangle, \left|11\right\rangle)^T,$$

$$\left|P_1\right\rangle = \frac{1}{2}(1, -ie^{i\theta_1}, -e^{i\theta_2}, ie^{i\theta_3})(\left|00\right\rangle, \left|01\right\rangle, \left|10\right\rangle, \left|11\right\rangle)^T,$$

$$\left|P_2\right\rangle = \frac{1}{2}(1, -e^{i\theta_1}, e^{i\theta_2}, -e^{i\theta_3})(\left|00\right\rangle, \left|01\right\rangle, \left|10\right\rangle, \left|11\right\rangle)^T,$$

$$\left|P_3\right\rangle = \frac{1}{2}(1, ie^{i\theta_1}, -e^{i\theta_2}, -ie^{i\theta_3})(\left|00\right\rangle, \left|01\right\rangle, \left|10\right\rangle, \left|11\right\rangle)^T,$$

$$\left|P_0'\right\rangle = \frac{1}{2}(1, -e^{i\theta_1}, -e^{i\theta_2}, e^{i\theta_3})(\left|00\right\rangle, \left|01\right\rangle, \left|10\right\rangle, \left|11\right\rangle)^T,$$

$$\left|P_1'\right\rangle = \frac{1}{2}(1, ie^{i\theta_1}, e^{i\theta_2}, ie^{i\theta_3})(\left|00\right\rangle, \left|01\right\rangle, \left|10\right\rangle, \left|11\right\rangle)^T,$$

$$\left|P_2'\right\rangle = \frac{1}{2}(1, e^{i\theta_1}, -e^{i\theta_2}, -e^{i\theta_3})(\left|00\right\rangle, \left|01\right\rangle, \left|10\right\rangle, \left|11\right\rangle)^T,$$

$$\left|P_3'\right\rangle = \frac{1}{2}(1, -ie^{i\theta_1}, e^{i\theta_2}, -ie^{i\theta_3})(\left|00\right\rangle, \left|01\right\rangle, \left|10\right\rangle, \left|11\right\rangle)^T,$$

$$\begin{aligned}\left|Q_h\right\rangle = \frac{1}{4}(&\left|0000\right\rangle + e^{i\beta_1 - \frac{h\pi i}{8}}\left|0001\right\rangle + e^{i\beta_2 - \frac{2h\pi i}{8}}\left|0010\right\rangle + e^{i\beta_3 - \frac{3h\pi i}{8}}\left|0011\right\rangle + e^{i\beta_4 - \frac{4h\pi i}{8}}\left|0100\right\rangle \\ &+ e^{i\beta_5 - \frac{5h\pi i}{8}}\left|0101\right\rangle + e^{i\beta_6 - \frac{6h\pi i}{8}}\left|0110\right\rangle + e^{i\beta_7 - \frac{7h\pi i}{8}}\left|0111\right\rangle + e^{i\beta_8 - \frac{8h\pi i}{8}}\left|1000\right\rangle \\ &+ e^{i\beta_9 - \frac{9h\pi i}{8}}\left|1001\right\rangle + e^{i\beta_{10} - \frac{10h\pi i}{8}}\left|1010\right\rangle + e^{i\beta_{11} - \frac{11h\pi i}{8}}\left|1011\right\rangle + e^{i\beta_{12} - \frac{12h\pi i}{8}}\left|1100\right\rangle \\ &+ e^{i\beta_{13} - \frac{13h\pi i}{8}}\left|1101\right\rangle + e^{i\beta_{14} - \frac{14h\pi i}{8}}\left|1110\right\rangle + e^{i\beta_{15} - \frac{15h\pi i}{8}}\left|1111\right\rangle),\end{aligned}$$

$$\begin{aligned}\left|Q_h'\right\rangle = \frac{1}{4}(&\left|0000\right\rangle - e^{i\beta_1 - \frac{h\pi i}{8}}\left|0001\right\rangle - e^{i\beta_2 - \frac{2h\pi i}{8}}\left|0010\right\rangle + e^{i\beta_3 - \frac{3h\pi i}{8}}\left|0011\right\rangle - e^{i\beta_4 - \frac{4h\pi i}{8}}\left|0100\right\rangle \\ &+ e^{i\beta_5 - \frac{5h\pi i}{8}}\left|0101\right\rangle + e^{i\beta_6 - \frac{6h\pi i}{8}}\left|0110\right\rangle - e^{i\beta_7 - \frac{7h\pi i}{8}}\left|0111\right\rangle - e^{i\beta_8 - \frac{8h\pi i}{8}}\left|1000\right\rangle \\ &+ e^{i\beta_9 - \frac{9h\pi i}{8}}\left|1001\right\rangle + e^{i\beta_{10} - \frac{10h\pi i}{8}}\left|1010\right\rangle - e^{i\beta_{11} - \frac{11h\pi i}{8}}\left|1011\right\rangle + e^{i\beta_{12} - \frac{12h\pi i}{8}}\left|1100\right\rangle \\ &- e^{i\beta_{13} - \frac{13h\pi i}{8}}\left|1101\right\rangle - e^{i\beta_{14} - \frac{14h\pi i}{8}}\left|1110\right\rangle + e^{i\beta_{15} - \frac{15h\pi i}{8}}\left|1111\right\rangle).\end{aligned}$$

Alice's and Bob's measurement results are $\{\left|A_m\right\rangle, \left|B_h\right\rangle \mid m = 0,1,2,3$ and $h = 0,1,...,15\}$. They will announce their measurement results through the classical channel.

[A-3] Charlie will measure his particle 1 in the basis $\{\left|0\right\rangle, \left|1\right\rangle\}$. Then, he announces his measurement result through the classical channel.

[A-4] If Charlie's measurement result is $\left|0\right\rangle$, Alice obtains $\left|\phi_{B4}\right\rangle$ by performing the unitary operations $U_{h,6} \otimes U_{h,8} \otimes U_{h,a} \otimes U_{h,c}$ and Bob obtains $\left|\phi_{A2}\right\rangle$ by carrying out the unitary operations $U_{m,3} \otimes U_{m,5}$, where

$$U_{h,6} \otimes U_{h,8} \otimes U_{h,a} \otimes U_{h,c} = \begin{pmatrix} 1 & 0 \\ 0 & e^{h\pi i} \end{pmatrix}_6 \otimes \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{h\pi i}{2}} \end{pmatrix}_8 \otimes \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{h\pi i}{4}} \end{pmatrix}_a \otimes \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{h\pi i}{8}} \end{pmatrix}_c , \tag{10}$$

$$U_{m,3} \otimes U_{m,5} = \begin{pmatrix} 1 & 0 \\ 0 & e^{m\pi i} \end{pmatrix}_3 \otimes \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{m\pi i}{2}} \end{pmatrix}_5 , m = 0,1,2,3. \tag{11}$$

If Charlie's measurement result is $|1\rangle$, the unitary operations of Alice and Bob are $U_{h,6}Z_{h,6} \otimes U_{h,8}Z_{h,8} \otimes U_{h,a}Z_{h,8} \otimes U_{h,c}Z_{h,c}$ and $U_{m,3}Z_{m,3} \otimes U_{m,5}Z_{m,5}$, respectively.

In the protocol, by using the technique of RSP, the security of quantum state transmission can be ensured. Moreover, Charlie as the controller, after Alice and Bob announcing their measurement results, measures his own particle and announces his result, which ensure the simultaneity, that is, Alice and Bob recover the desired state at the same time. The classical communication costs are 7 cbits. Furthermore, the success probability of our protocol can be calculated as

$$P_{success} = \sum_{j=0}^{1} \sum_{m=0}^{3} \sum_{h=0}^{15} [(\frac{1}{2})^2 (\frac{1}{4})^2 (\frac{1}{\sqrt{2}})^2] = 1. \tag{12}$$

Therefore, through implementing our protocol, the desired states can be transmitted securely, simultaneously and deterministically.

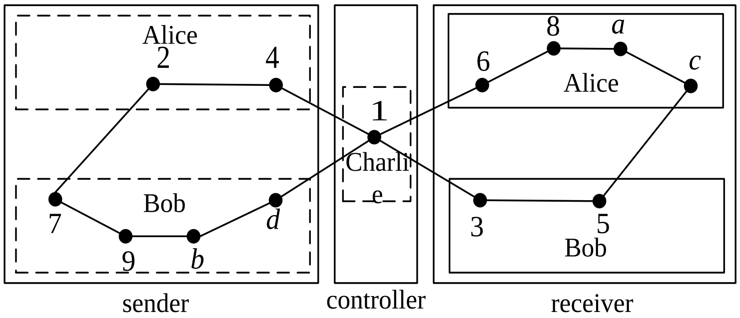

Figure 1: The process of our CBRSP protocol. Each solid point represents a particle, and the solid line represents an entanglement between particles. Dotted rectangle indicates the measurement. Solid rectangle indicates the unitary operation

3 Discussions

In this section, we discuss the efficiency and the necessary operations of our proto-col at first. Furthermore, some comparisons with other protocols are given.

3.1 Efficiency

The efficiency [Yuan, Liu, Zhang et al. (2008)] is an important factor in measuring protocol performance, which can be calculated as

$$\eta = \frac{q_s}{q_u + b_t}, \tag{13}$$

where q_s denotes the number of qubits that consist of the quantum information to be prepared, q_u is the number of the qubits that is used as the quantum channel and b_t is the classical bits transmitted.

Therefore, the efficiency of our protocol is calculated as

$$\eta = \frac{6}{13+7} = 30\%. \tag{14}$$

3.2 The necessary operations

In our protocol, the necessary operations only used in constructing the quantum channel are seven *H* operations and twelve *CNOT* operations. Furthermore, no additional operations are needed.

3.3 Comparisons with other protocols

In the previous protocols [Cao and Nguyen (2013); Sharma, Shukla, Banerjee et al. (2015); Peng, Bai and Mo (2015); Wang and Mo (2017); Zhang, Zha, Duan et al. (2016a, 2016b); Sang (2017); Song, Ni, Wang et al. (2017); Wu, Zha and Yang (2018); Sang and Nie (2017); Fang and Jiang (2018)], they only achieved symmetric bidirectional transmission of single-qubit state or the asymmetric transmission of single-and two-qubit state. Even in Ma et al. [Ma, Chen, Li et al. (2017)], they achieved the asymmetric bidirectional preparation of single-and four-qubit cluster-type states. However, in fact, they only transmit the single-and two-qubit state. And one of the receivers needs two local auxiliary qubits and auxiliary operations to recover the four-qubit cluster-type state. The results of comparison with the pervious asymmetric protocols are given in Tab. 1.

Firstly, by using seven *H* operations and twelve *CNOT* operations, we give the construction of the quantum channel while the previous protocols have not given. Secondly, the sender or receiver of some protocols [Cao and Nguyen (2013); Sharma, Shukla, Banerjee et al. (2015); Zhang, Zha, Duan et al. (2016a, 2016b); Sang (2017); Song, Ni, Wang et al. (2017); Fang and Jiang (2018); Ma, Chen, Li et al. (2017)] needs the help of auxiliary particles and additional operations to complete the bidirectional task, which are not needed in our protocol. Thirdly, similarly, both protocols prepare four-qubit states, while the classical communication costs of our protocol are fewer than that in Ref. [Ma, Chen, Li et al. (2017)]. Last but not the least, since our protocol can realize the bidirectional transmission of two-and four-qubit equatorial states, our protocol is more efficient than other protocols.

The point-to-point quantum communication must be turned to the multi-party quantum network communication. These have a wide range of research meanings in some network structures [Guo, Zhang, Liu et al. (2017); Pang, Liu, Zhou et al. (2017); Li, Wang, Li et al. (2018); Shen, Song, Li et al. (2018)]. As regards the quantum networks, the feasibility and construction have been fully verified theoretically [Dong, Zhang, Zhang et al. (2014); Jiang, Jiang and Ling (2014); Xu, Chen, Li et al. (2015); Li, Chen, Xu et al. (2015)]. Our scheme do not need the auxiliary resources and have relatively high efficiency, so it can be easily incorporated into the design of quantum network communication.

Table 1: Comparisons with the pervious asymmetric protocols

Protocol	Quantum Channel	Prepared State	Auxiliary Particles	Additional Operations	CCCs	Efficiency (%)
[Song, Ni, Wang et al. (2017)]	Four-qubit cluster state+EPR	Single- and Two-qubit	1	1*CNOT*	6	23.08%
[Sang and Nie (2017)]	Seven-qubit Entangled State	Single- and Two-qubit	0	0	5	25%
[Fang and Jiang (2018)]	Seven-qubit Entangled State	Single- and Two-qubit	2	2*CNOTs*	7	18.75%
[Fang and Jiang (2018)]	Seven-qubit Entangled State	Two- and Single-qubit	1	1*CNOT*	7	20%
[Ma, Chen, Li et al. (2017)]	Ten-qubit Entangled State	Single- and Four-qubit	5	7*CNOTs*	8	13.04%
Ours	Thirteen-qubit Entangled State	Two- and Four-qubit	0	0	7	30%

Where CCCs=classical communication costs.

4 Discussions

In summary, an asymmetric protocol is proposed for controlled bidirectional quantum state transmission. In this protocol, by using the thirteen-qubit entangled state as the quantum channel, we can realize the bidirectional transmission of two-and four-qubit equatorial states simultaneously, securely and deterministically. In the end, we analyze the performance of the protocol and describe some comparisons with other protocols, including the efficiency, the necessary operations and the classical communication costs. The results show that our protocol has better performance.

Acknowledgement: Project supported by NSFC (Grant Nos. U1836205, 61702040), the Major Scientific and Technological Special Project of Guizhou Province (Grant No. 20183001), the Foundation of Guizhou Provincial Key Laboratory of Public Big Data (Grant No. 2018BDKFJJ016), the Foundation of State Key Laboratory of Public Big Data (Grant No. 2018BDKFJJ018) and Beijing Natural Science Foundation (Grant No. 4174089).

References

An, N. B. (2010): Joint remote state preparation via W and W-type states. *Optical Communication*, vol. 283, pp. 4113-4117.

Bennett, C. H.; Brassard, G.; Crépeau, C.; Jozsa, R.; Peres, A. et al. (1993): Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Physical Review Letter*, vol. 70, pp. 1895.

Cavaillès, A. I.; Le Jeannic, H.; Raskop, J.; Huang, K.; Laurat, J. (2017): Remote state preparation and quantum steering based on optical hybrid entanglement. *Quantum Information and Measurement.*

Chen, X. B.; Ma, S. Y.; Su, Y.; Zhang, R.; Yang, Y. X. (2012): Controlled remote state preparation of arbitrary two and three qubit states via the Brown state. *Quantum Information Processing*, vol. 11, no. 6, pp. 1653-1667.

Choudhury, B. S.; Dhara, A. (2015): Joint remote state preparation for two-qubit equatorial states. *Quantum Information Processing*, vol. 14, no. 1, pp. 373-379.

Cao, T. B.; Nguyen, B. A. (2013): Deterministic controlled bidirectional remote state preparation. *Advances in Natural Sciences: Nanoscience and Nanotechnology*, vol. 5, no. 1, 015003.

Chen, Q. Q.; Xia, Y.; Song, J.; An, N. B. (2010): Joint remote state preparation of a W-type state via W-type states. *Physics Letters A*, vol. 374, no. 44, pp. 4483-4487.

Chen, X. B.; Tang, X.; Xu, G.; Dou, Z.; Chen, Y. L. et al. (2018): Cryptanalysis of secret sharing with a single d-level quantum system. *Quantum Information Processing*, vol. 17, no. 9, pp. 225.

Chen, X. B.; Sun, Y. R.; Xu, G.; Jia, H. Y.; Qu, Z. et al. (2017): Controlled bidirectional remote preparation of three-qubit state. *Quantum Information Processing*, vol. 16, no. 10, pp. 244.

Chen, X. B.; Su, Y.; Xu, G.; Sun, Y.; Yang, Y. X. (2014): Quantum state secure transmission in network communications. *Information Sciences*, vol. 276, no. 34, pp. 363-376.

Devetak, I.; Berger, T. (2001): Low-entanglement remote state preparation. *Physical Review Letters*, vol. 87, no. 19, 197901.

Dong H.; Zhang Y.; Zhang Y.; Yin, B. (2014): Generalized bilinear differential operators, binary bell polynomials, and exact periodic wave solution of boiti-leon-manna-pempinelli equation. *Abstract and Applied Analysis. Hindawi*, vol. 2014.

Fang, S. H.; Jiang, M. (2018): A novel scheme for bidirectional and hybrid quantum information transmission via a seven-qubit state. *International Journal of Theoretical Physics*, vol. 57, no. 2, pp. 523-532.

Fortes, R.; Rigolin, G. (2017): Probabilistic quantum teleportation via thermal entanglement. *Physical Review A*, vol. 96, no. 2, 022315.

Guo, R.; Zhang, Z.; Liu, X.; Lin, C. (2017): Existence, uniqueness, and exponential stability analysis for complex-valued memristor-based BAM neural networks with time delays. *Applied Mathematics and Computation*, vol. 311, pp. 100-117.

Huang, L.; Zhao, H. X. (2017): Controlled remote state preparation of an arbitrary two-qubit state by using GHZ states. *International Journal of Theoretical Physics*, vol. 56, no. 3, pp. 678-682.

Jiang, T.; Jiang, Z.; Ling, S. (2014): An algebraic method for quaternion and complex least squares coneigen-problem in quantum mechanics. *Applied Mathematics and Computation*, vol. 249, pp. 222-228.

Kiktenko, E. O.; Popov, A. A.; Fedorov, A. K. (2016): Bidirectional imperfect quantum teleportation with a single Bell state. *Physical Review A*, vol. 93, no. 6, 062305.

Lin, S.; Guo, G. D.; Xu, Y. Z.; Sun, Y.; Liu, X. F. (2016): Cryptanalysis of quantum secret sharing with *d*-level single particles. *Physical Review A*, vol. 93, no. 6, 062343.

Li, X. H.; Shohini, G. (2017): Analysis of control power in controlled remote state preparation schemes. *International Journal of Theoretical Physics*, vol. 56, no. 3, pp. 667-677.

Li, Y. H.; Li, X. L.; Sang, M. H.; Nie, Y. Y.; Wang, Z. S. (2013): Bidirectional controlled quantum teleportation and secure direct communication using five-qubit entangled state. *Quantum Information Processing*, vol. 12, no. 12, pp. 3835-3844.

Luo, M. X.; Chen, X. B.; Yang, Y. X.; Niu, X. X. (2012): Experimental architecture of joint remote state preparation. *Quantum Information Processing*, vol. 11, no. 3, pp. 751-767.

Luo, M. X.; Chen, X. B.; Ma, S. Y.; Yang, Y. X.; Hu, Z. M. (2010): Deterministic remote preparation of an arbitrary W-class state with multiparty. *Journal of Physics B: Atomic, Molecular and Optical Physics*, vol. 43, no. 6, 065501.

Liu, L. L.; Hwang, T. (2014): Controlled remote state preparation protocols via AKLT states. *Quantum Information Processing*, vol. 13, no. 7, pp. 1639-1650.

Li, L.; Wang, Z.; Li, Y.; Shen, H.; Lu, J. (2018): Hopf bifurcation analysis of a complex-valued neural network model with discrete and distributed delays. *Applied Mathematics and Computation*, vol. 330, pp. 152-169.

Li, J.; Chen, X. B.; Xu, G.; Yang, Y. X.; Li, Z. P. (2015): Perfect quantum network coding independent of classical network solutions. *IEEE Communications Letters*, vol. 19, no. 2, pp. 115-118.

Liu, W. J.; Gao, P. P.; Yu, W. B.; Qu, Z. G.; Yang, C. N. (2018): Quantum relief algorithm. *Quantum Information Processing*, vol. 17, no. 10, pp. 280.

Liu, W. J.; Xu, Y.; Yang, C. N.; Gao, P. P.; Yu, W. B. (2018): An efficient and secure arbitrary n-party quantum key agreement protocol using bell states. *International Journal of Theoretical Physics*, vol. 57, no. 1, pp. 195-207.

Liu, W. J.; Wang, H. B.; Yuan, G. L.; Xu, Y.; Chen, Z. Y. et al. (2016): Multiparty quantum sealed-bid auction using single photons as message carrier. *Quantum Information Processing*, vol. 15, no. 2, pp. 869-879.

Ma, P. C.; Chen, G. B.; Li, X. W.; Zhan, Y. B. (2017): Asymmetric bidirectional controlled remote preparation of an arbitrary four-qubit cluster-type state and a single-qubit state. *Quantum Information Processing*, vol. 16, no. 12, pp. 308.

Pati, A. K. (2000): Minimum classical bit for remote preparation and measurement of a qubit. *Physical Review A*, vol. 63, no. 1, 014302.

Pang, Z.; Liu, G.; Zhou, D.; Sun, D. (2017): Data-based predictive control for networked nonlinear systems with packet dropout and measurement noise. *Journal of Systems Science and Complexity*, vol. 30, pp. 1072-1083.

Peng, J. Y.; Bai, M. Q.; Mo, Z. W. (2015): Bidirectional controlled joint remote state preparation. *Quantum Information Processing*, vol. 14, no. 11, pp. 4263-4278.

Qu, Z. G.; Wu, S.; Wang, M.; Sun, L.; Wang, X. (2017): Effect of quantum noise on deterministic remote state preparation of an arbitrary two-particle state via various quantum entangled channels. *Quantum Information Processing*, vol. 16, no. 12, pp. 306.

Qu, Z. G.; Cheng, Z.; Liu, W.; Wang, X. (2018): A novel quantum image steganography algorithm based on exploiting modification direction. *Multimedia Tools and Applications*, pp. 1-21.

Qu, Z. G.; Chen, S.; Ji, S.; Ma, S.; Wang, X. (2018): Anti-noise bidirectional quantum steganography protocol with large payload. *International Journal of Theoretical Physics*, vol. 57, no. 6, pp. 1903-1927.

Rådmark, M.; Wieśniak, M.; Żukowski, M.; Bourennane, M. (2013): Experimental multilocation remote state preparation. *Physical Review A*, vol. 88, no. 3, 032304.

Shukla, C.; Thapliyal, K.; Pathak, A. (2017): Hierarchical joint remote state preparation in noisy environment. *Quantum Information Processing*, vol. 16, no. 8, pp. 205.

Sharma, V.; Shukla, C.; Banerjee, S.; Pathak, A. (2015): Controlled bidirectional remote state preparation in noisy environment: a generalized view. *Quantum Information Processing*, vol. 14, no. 9, pp. 3441-3464.

Sang, Z. W. (2017): Bidirectional controlled quantum information transmission by using a five-qubit cluster state. *International Journal of Theoretical Physics*, vol. 56, no. 11, pp. 3400-3404.

Song, Y.; Ni, J. L.; Wang, Z. Y.; Lu, Y.; Han, L. F. (2017): Deterministic bidirectional remote state preparation of a-and symmetric quantum states with a proper quantum channel. *International Journal of Theoretical Physics*, vol. 56, no. 10, pp. 3175-3187.

Sang, M. H.; Nie, L. P. (2017): Asymmetric bidirectional controlled quantum information transmission via seven-particle entangled state. *International Journal of Theoretical Physics*, vol. 56, no. 11, pp. 3638-3641.

Sisodia, M.; Shukla, A.; Thapliyal, K.; Pathak, A. (2017): Design and experimental realization of an optimal scheme for teleportation of an *n*-qubit quantum state. *Quantum Information Processing*, vol. 16, no. 12, pp. 292.

Shen, H.; Song, X. N.; Li, F.; Wang, Z.; Chen, B. (2018): Finite-time L2-L∞ filter design for networked Markov switched singular systems: a unified method. *Applied Mathematics and Computation*, vol. 321, no. 15, pp. 450-462.

Tan, X. Q; Li, X. C.; Yang, P. (2018): Perfect Quantum Teleportation via Bell States. *Computers, Materials & Continua*, vol. 57, no. 3, pp. 495-503.

Thapliyal, K.; Verma, A.; Pathak, A. (2015): A general method for selecting quantum channel for bidirectional controlled state teleportation and other schemes of controlled quantum communication. *Quantum Information Processing*, vol. 14, no. 12, pp. 4601-4614.

Thapliyal, K.; Pathak, A. (2015): Applications of quantum cryptographic switch: various tasks related to controlled quantum communication can be performed using Bell states and permutation of particles. *Quantum Information Processing*, vol. 14, no. 7, pp. 2599-2616.

Wei, J.; Shi, L.; Zhu, Y.; Xue, Y.; Xu, Z. et al. (2018): Deterministic remote preparation of arbitrary multi-qubit equatorial states via two-qubit entangled states. *Quantum Information Processing*, vol. 17, no. 3, pp. 70.

Wang, C.; Deng, F. G.; Li, Y. S.; Liu, X. S.; Long, G. L. (2005): Quantum secure direct communication with high-dimension quantum super dense coding. *Physical Review A*, vol. 71, no. 4, 044305.

Wang, C.; Zeng, Z.; Li, X. H. (2015): Controlled remote state preparation via partially entangled quantum channel. *Quantum Information Processing*, vol. 14, no. 3, pp. 1077-1089.

Wu, H.; Zha, X. W.; Yang, Y. Q. (2018): Controlled bidirectional hybrid of remote state preparation and quantum teleportation via seven-qubit entangled state. *International Journal of Theoretical Physics*, vol. 57, no. 1, pp. 28-35.

Wei, Z. H.; Chen, X. B.; Niu, X. X.; Yang, Y. X. (2015): The quantum steganography protocol via quantum noisy channels. *International Journal of Theoretical Physics*, vol. 54, no. 8, pp. 2505-2515.

Wang, X. Y.; Mo, Z. W. (2017): Bidirectional controlled joint remote state preparation via a seven-qubit entangled state. *International Journal of Theoretical Physics*, vol. 56, no. 4, pp. 1052-1058.

Xu, G.; Chen, X. B.; Li, J.; Wang, C.; Yang, Y. X. et al. (2015): Network coding for quantum cooperative multicast. *Quantum Information Processing*, vol. 14, no. 11, pp. 4297-4322.

Xu, G.; Chen, X. B.; Duo, Z.; Yang, Y. X.; Li, Z. (2015): A novel protocol for multiparty quantum key management. *Quantum Information Processing*, vol. 14, no. 8, pp. 2959-2980.

Xu, G.; Chen, X. B.; Dou, Z.; Li, J.; Liu, X. et al. (2016): Novel criteria for deterministic remote state preparation via the entangled six-qubit state. *Entropy*, vol. 18, pp. 267.

Yuan, H.; Liu, Y. M.; Zhang, W.; Zhang, Z. J. (2008): Optimizing resource consumption, operation complexity and efficiency in quantum-state sharing. *Journal of Physics B: Atomic, Molecular and Optical Physics*, vol. 41, no. 14, pp. 145506.

Zhang, D.; Zha, X.; Duan, Y.; Wei, Z. H. (2016a): Deterministic controlled bidirectional remote state preparation via a six-qubit maximally entangled state. *International Journal of Theoretical Physics*, vol. 55, no. 1, pp. 440-446.

Zhang, D.; Zha, X.; Duan, Y.; Yang, Y. (2016b): Deterministic controlled bidirectional remote state preparation via a six-qubit entangled state. *Quantum Information Processing*, vol. 15, no. 5, pp. 2169-2179.

Zhang, Q.; Goebel, A.; Wagenknecht, C.; Chen, Y. A.; Zhao, B. et al. (2006): Experimental quantum teleportation of a two-qubit composite system. *Nature Physics*, vol. 2, no. 10, pp. 678.