

Forced Vibrations of a System Consisting of a Pre-strained Highly Elastic Plate under Compressible Viscous Fluid Loading

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Abstract: The forced vibration of the system consisting of the pre-stretched plate made of highly-elastic material and half-plane filled by barotropic compressible Newtonian viscous fluid is considered. It is assumed that this forced vibration is caused by the lineal located time-harmonic force acting on the free face plane of the plate. The motion of the pre-stretched plate is written by utilizing of the linearized exact equations of the theory of elastic waves in the initially stressed bodies, but the motion of the compressible viscous fluid is described by the linearized Navier-Stokes equations. The elastic relations of the plate material are described with the use of the harmonic potential. Moreover, it is assumed that the velocities and stresses of the constituents are continuous on the contact plane between the plate and fluid. The dimensionless parameters which characterize the compressibility, viscosity of the fluid and elastic constants of the plate material are introduced. The plane strain state in the plate is considered and the corresponding boundary- and contact-value problem is solved by employing exponential Fourier transformation with respect to the coordinate directed along the interface line and the inverse of this transformation is determined numerically by employing the Sommerfeld contour. Numerical results on the interface stresses and velocities and the influence of the problem parameters such as initial strains and thickness of the plate, the compressibility and viscosity of the plate, as well the magnitude of the frequency of the external forces on these results are presented and discussed. Numerical results are examined in the case where the fluid is Glycerin and the values of the elastic constants which enter into the mentioned above harmonic potential and the density of the plate material are taken as Lamé's constants and density of

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the Plexiglass (Lucite).

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1 Introduction

Investigations of problems related to the dynamics of plate-fluid interaction have great significance in the theoretical and application sense in aerospace, nuclear, naval, chemical and biological engineering. The first attempt in this field was made by Lamb (1921) in which vibration of a circular elastic “baffled” plate in contact with still water were considered. It was assumed that this plate is clamped all around and placed in a matching circular aperture within an infinite rigid plane wall. The investigations were made by the use of the so-called “non-dimensional added virtual mass incremental” (NAVMI) method, according to which, it is assumed that the modes of vibration of the plate in contact with still water are the same as those in a vacuum, and the natural frequency is determined by the use of the Rayleigh quotient. In this case it is supposed that the squares of the natural frequencies of the plate are equal to the ratio between the maximum potential energy of the plate and the sum of the kinetic energies of both the plate and the fluid. Later this method was employed in many related investigations such as in papers by Kwak and Kim (1991), Fu and Price (1987), Zhao and Yu (2012) and in many others listed in these papers. Up to now there are also the investigations carried out without employing the NAVMI method. For instance, in a paper by Tubaldi and Armabili (2013) the vibration and stability of the rectangular plate immersed in axial liquid flow was studied without employing the NAVMI method, and the Galerkin method was applied to determine the expression of the flow perturbation potential. Then the Rayleigh-Ritz method was used to discretize the system.

Investigations carried out in a paper by Charman and Sorokin (2005) and others listed therein were also made without employing the NAVMI method. Note that in this paper the forced bending vibration of an infinite plate in contact with compressible (acoustic) inviscid fluid, where this fluid occupies a half-plane (half-space), was considered. This paper gives asymptotic analyses of the sound and vibration in the metal plate and compressible inviscid fluid system.

The other aspect of investigations related to the plate-fluid interaction regards wave propagation problems. Investigations carried out in a paper by Sorokin and Chubinskij (2008) and others listed therein can be taken as examples of this. It should be noted that before this paper the problems of time harmonic linear wave propagation in elastic structure-fluid systems were investigated within the framework of the theory of compressible inviscid fluid. A list of these studies and a review is

given in the aforementioned paper by Sorokin and Chubinskij (2008). At the same time, the role of fluid viscosity in wave propagation in the plate-fluid system was first investigated in this paper. However, in this paper and all the papers indicated above, the equations of motion of the plate were written within the scope of the approximate plate theories by the use of various types of hypotheses such as the Kirchhoff hypotheses for plates. Consequently, the use of the approximate plate theories in these investigations decreases significantly the analyzed range of wave modes and their corresponding dispersion curves. It is evident that in many cases (for instance, in the cases where the wave length is less significant than the thickness of the plate) more accurate results in the qualitative and quantitative sense, can be obtained by employing the exact equations for describing plate motion. Moreover, in the foregoing investigations (except the paper by Zhao and Yu (2012)) the initial strains (or stresses) in plates, which can be one of their characteristic particularities, are not taken into account. These two characteristics, namely the use of the exact equations of plate motion and the existence of initial stresses in the plate are taken into consideration in a paper by Bagnò et al. (1994) and others, a review of which is given in a survey paper by Bagnò and Guz (1997). Note that in these papers, in studying wave propagation in pre-stressed plate + compressible viscous fluid systems, the motion of the plate was written within the scope of the so-called three-dimensional linearized theory of elastic waves in initially stressed bodies (TLTEWISB). However, the motion of the viscous fluid was written within the scope of the linearized Navier-Stokes equations. Detailed consideration of related results was made in the monograph by Guz (2009).

However, up to now within this framework there is not an investigation related to the forced vibration of the pre-strained plate + compressible viscous fluid system. In the present paper the attempt is made in this field and two-dimensional (plane-strain state) problem on the forced vibration of the pre-strained plate made of highly-elastic material + compressible viscous fluid system is studied. The motion of the plate is described by utilizing of the TLTEWISB, and the motion of the fluid by utilizing of the linearized Navier-Stokes equations. Numerical results on the velocity and stress distributions on the plate-fluid interface and the influence of the problem parameters such as initial strains of the plate, the thickness of the plate, the frequency of the external force etc. on these distributions are presented and discussed. These results are obtained in the case where the fluid is Glycerin and the values of the elastic constants which enter into the mentioned above harmonic potential and the density of the plate material are taken as Lamé's constants and density of the Plexiglass (Lucite). Consequently, in the case where the initial strains is absent in the plate the numerical results relate to the hydro-elastic system consisting of the Plexiglass and Glycerin.

Note that corresponding problems related to the pre-strained plate + elastic half-space were studied in papers by Akbarov (2006 a, b, 2013a) Akbarov and Guler (2007), Akbarov et al (2005), Akbarov and Ilhan (2009, 2010), Akbarov et al (2013) and Emiroglu et al (2009).

2 Formulation of the problem and governing field equations

Consider a system consisting of an initially stretched plate-layer which is in contact with the half-space occupied by the compressible Newtonian viscous fluid (Fig.1). Assume that the thickness of the plate in the natural state (i.e. before the initial stretching of that) is h . We consider separately the governing field equations for the plate-layer and for the compressible Newtonian viscous fluid

2.1 Governing field equations for the plate-layer.

In the natural state, we determine positions of the points of the layer by the Lagrangian coordinates in the Cartesian system of coordinates $Ox_1x_2x_3$. Suppose that the layer has infinite length in the directions of the Ox_1 and Ox_3 axes. The Ox_3 axis extends along a direction perpendicular to the plane Ox_1x_2 in Fig. 1 and therefore is not shown in this figure.

We propose that the layer made of highly-elastic material, before being contacted with fluid, be stretched along the Ox_1 axis direction and as result of this stretching the homogeneous finite strain state appears in the layer. Namely this strain state is called the initial strain state in the layer. Note that the initial strains are caused by the static forces acting in the Ox_1 axis direction at infinity and the action of these forces continues all further dynamic process.

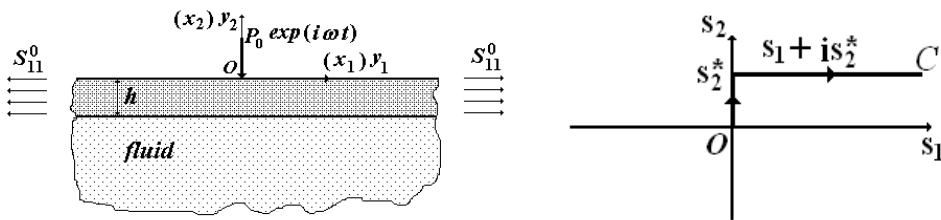


Figure 1: The sketch of the system under consideration (a) and Sommerfeld contour (b).

With the initial state of the layer we associate the Lagrangian Cartesian system of coordinates $Oy_1y_2y_3$ and suppose that the origin of the system coincides with the origin of the system $Ox_1x_2x_3$, and the coordinate axes Oy_1 , Oy_2 and Oy_3 coincide

with the coordinate axes Ox_1, Ox_2 and Ox_3 , respectively. Assuming that the material of the layer is compressible, the elastic relations are given through the harmonic potential.

Below the values related to the initial state are denoted by upper index 0. Thus, according to the foregoing, the initial state in the layer can be determined as follows.

$$u_1^0 = (\lambda_1 - 1)x_1, \quad u_2^0 = (\lambda_2 - 1)x_2, \quad u_3^0 = 0 \quad \lambda_1 = const_1 \neq 1, \quad \lambda_2 = const_2 \neq 1, \quad \lambda_3 = 1, \quad y_1 = \lambda_1 x_1, y_2 = \lambda_2 x_2, \quad y_3 = x_3, \tag{1}$$

where u_k^0 ($k = 1, 3$) is a component of the displacement vector in the layer in the initial strain state and λ_k is an elongation factor which characterizes the change in the length of the line element in the Ox_k axis direction. This parameter determined by the expression $\lambda_k = \sqrt{1 + 2\varepsilon_k}$, where ε_k is the k -th principal value of the Green's strain tensor. The expression of the components of this tensor through the components of the displacement vector will be given below.

Within this consider a motion of the layer in the case where on the free face plane of that the line-located normal time harmonic force acts. This consideration will be made by the use of coordinates associated with the initial state, i.e. by the use of coordinates y_k ($k = 1, 3$), in the framework of the three-dimensional linearized theory of elastic waves in initially stressed bodies (TLTEWISB). In the construction of the field equations of the TLTEWISB, one considers two states of a deformable solid. The first is regarded as the initial or unperturbed state and the second is a perturbed state with respect to the unperturbed one. By the "state of a deformable solid" both motion and equilibrium (as a particular case of motion) is meant. It is assumed that all values in a perturbed state can be represented as a sum of the values in the initial state and the perturbations. The latter is also assumed to be small in comparison with the corresponding values in the initial state. It is also assumed that both initial (unperturbed) and perturbed states are described by the equations of nonlinear solid mechanics. Owing to the fact that the perturbations are small, the relationships for the perturbed state in the vicinity of appropriate values for the unperturbed state are linearized, and then the relations for the perturbed state are subtracted from them. The results is the equations of the TLTEWISB. The general problems of the TLTEWISB have been elaborated in many investigations such as Biot (1965), Guz (2004), Truestell and Noll (1965) and others.

Thus, the following are the basic relations of the TLTEWISB for the compressible body under the plane-strain state in the Oy_1y_2 plane.

The equation of motion is

$$\frac{\partial Q_{ij}}{\partial y_i} = \rho \frac{\partial^2 u_j}{\partial t^2}, \tag{2}$$

and mechanical relations are

$$Q_{ij} = \omega_{ij\alpha\beta} \frac{\partial u_\alpha}{\partial y_\beta}, \tag{3}$$

where $i, j, \alpha, \beta = 1, 2$ and Einstein summation rule is employed with respect to the repeated indices in (2) and (3). Moreover, in equation (2) and (3) the following notation is used: Q_{ij} are the components of the perturbations of the Kirchhoff non-symmetric stress tensor related to the areas of the initial state, u_j are the components of the perturbations of the displacement vector, and ρ is the density also related to the volume of the initial state. Expressions for determination of the components $\omega_{ij\alpha\beta}$ will be determined below. Note that these components are determined through the initial strain state (1) and through the elastic potential. As has noted above, in the present work the elastic relations of the layer's material are determined by harmonic potential. This potential is given as follows:

$$\varphi = \frac{1}{2} \lambda (s_1)^2 + \mu s_2 \tag{4}$$

where λ and μ are the mechanical constants of the material and

$$s_1 = (\sqrt{1+2\varepsilon_1} - 1) + (\sqrt{1+2\varepsilon_2} - 1) + (\sqrt{1+2\varepsilon_3} - 1),$$

$$s_2 = (\sqrt{1+2\varepsilon_1} - 1)^2 + (\sqrt{1+2\varepsilon_2} - 1)^2 + (\sqrt{1+2\varepsilon_3} - 1)^2. \tag{5}$$

In (5) ε_i ($i = 1, 2, 3$) are the principal values of the Green's strain tensor.

Let us consider briefly the definition of the stress and strain tensors in the large elastic deformation theory which are used in the present investigation. For this purpose we use the Lagrange coordinates x_i ($i = 1, 2, 3$) in the Cartesian system of coordinates $Ox_1x_2x_3$ and the position of the points after and before deformations we determine by the vectors \mathbf{r}^* and \mathbf{r} respectively where $\mathbf{r}^* = \mathbf{r} + \mathbf{u}$. Here $\mathbf{u} = u_i \mathbf{g}_i$ is a displacement vector expressed by the unit basic vectors \mathbf{g}_i . Taking the relations $d\mathbf{r}^* \cdot d\mathbf{r}^* = d\mathbf{r} \cdot d\mathbf{r} + 2d\mathbf{r} \cdot d\mathbf{u} + d\mathbf{u} \cdot d\mathbf{u}$ (here the symbol "." means the scalar product of the vectors), $d\mathbf{u} \cdot d\mathbf{u} = (\partial u_k / \partial x_i)(\partial u_k / \partial x_j) dx_i dx_j$ and $2d\mathbf{r} \cdot d\mathbf{u} = 2(\partial u_k / \partial x_i) dx_k dx_j$ into account, it can be written that $d\mathbf{r}^* \cdot d\mathbf{r}^* - d\mathbf{r} \cdot d\mathbf{r} = 2\varepsilon_{ij} dx_i dx_j$, where

$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_n}{\partial x_i} \frac{\partial u_n}{\partial x_j} \right). \tag{6}$$

This is a component of the Green's strain tensor $\tilde{\varepsilon}$ which is symmetric.

Let us consider the definition of the Kirchhoff stress tensor. The use of various types of stress tensors in the large (finite) elastic deformation theory is connected

with the reference of the components of these tensors to the unit area of the relevant surface elements in the deformed or un-deformed state, because, in contrast to the linear theory of elasticity, in the finite elastic deformation theory the difference between the areas of the surface elements taken before and after deformation must be accounted for in the derivation of the equation of motion and under satisfaction of the boundary conditions. According to the aim of the present investigation, we here consider two types of stress tensors denoted by \tilde{q} and \tilde{S} the components of which refer to the unit area of the relevant surface elements in the un-deformed state, but act on the surface elements in the deformed state. The components S_{ij} of the stress tensor \tilde{S} are determined through the strain energy potential $\varphi = \varphi(\epsilon_{11}, \epsilon_{22}, \dots, \epsilon_{33})$, where ϵ_{ij} is a component of the Green's strain tensor (6), by the use of the following expression:

$$S_{ij} = \frac{1}{2} \left(\frac{\partial}{\partial \epsilon_{ij}} + \frac{\partial}{\partial \epsilon_{ji}} \right) \varphi(\epsilon_{11}, \epsilon_{22}, \dots, \epsilon_{33}). \tag{7}$$

The components q_{ij} of the stress tensor \tilde{q} are determined by the expression

$$q_{ij} = \left(\delta_k^j + \frac{\partial u_j}{\partial x_k} \right) S_{ik}. \tag{8}$$

Here δ_k^j is the Kronecker symbol. The stress tensor \tilde{q} with components determined by expressions (7) and (8) is called the Kirchhoff stress tensor. According to expressions (6)-(8), the stress tensor \tilde{S} is symmetric, but the Kirchhoff stress tensor \tilde{q} is non-symmetric. Thus, with this we restrict ourselves to consideration of the definition of the stress and strain tensors in the finite elastic deformation theory. These definitions are given without any restriction related to the association of the selected coordinate systems to the natural or initial state. However, in using the coordinate system associated system with the initial deformed state, the initial strain state can be taken as an "un-deformed" state in the foregoing definitions.

Now we attempt to attain the Eq. (3) and the expressions of the components $\omega_{ij\alpha\beta}$ by employing the linearization procedure. Thus, according to (1), (4)-(8), we attain that

$$\begin{aligned} S_{11}^0 &= [\lambda(\lambda_1 + \lambda_2 - 2) + 2\mu(\lambda_1 - 1)](\lambda_1), \\ S_{22}^0 &= [\lambda(\lambda_1 + \lambda_2 - 2) + 2\mu(\lambda_2 - 1)](\lambda_2) = 0, \\ S_{12}^0 &= 0. \end{aligned} \tag{9}$$

It follows from the second expression in (9) that

$$\lambda_2 = [2\mu - \lambda(\lambda_1 - 2)] / (\lambda + 2\mu)^{-1}. \tag{10}$$

In this way, for a selected material the magnitude of the initial strains and the initial stresses in the layer can be determined through λ_1 only. In this case the perturbation of the components of the non-symmetric Kirchhoff stress tensor q_{ij} (denoted by q'_{ij}) related to the areas of the natural state are determined by the following expression:

$$q'_{ij} = \left(\delta_k^j + \frac{\partial u_j^0}{\partial x_k} \right) S'_{ik} + S_{ik}^0 \frac{\partial u_j}{\partial x_k}, \tag{11}$$

where S'_{ik} is a perturbation of the components of the foregoing stress tensor \tilde{S} .

By linearization of Eq. (7), the following relation is obtained for these components:

$$S'_{in} = \left\{ \frac{1}{4} \left(\delta_k^\beta + \frac{\partial u_\beta^0}{\partial x_k} \right) \left(\frac{\partial}{\partial \varepsilon_{k\beta}^0} + \frac{\partial}{\partial \varepsilon_{\beta k}^0} \right) \left(\frac{\partial}{\partial \varepsilon_{in}^0} + \frac{\partial}{\partial \varepsilon_{ni}^0} \right) \varphi^0 \right\} \frac{\partial u_\alpha}{\partial x_\beta}. \tag{12}$$

Using the relations

$$\begin{aligned} Q_{11} dy_2 dy_3 &= q'_{11} dx_2 dx_3, & Q_{22} dy_1 dy_3 &= q'_{22} dx_1 dx_3, & Q_{12} dy_2 dy_3 &= q'_{12} dx_2 dx_3, \\ Q_{21} dy_1 dy_3 &= q'_{21} dx_1 dx_3, & dy_1 &= \lambda_1 dx_1, & dy_2 &= \lambda_2 dx_2 \Rightarrow Q_{11} = q'_{11} / \lambda_2, \\ Q_{22} &= q'_{22} / \lambda_1, & Q_{12} &= q'_{12} / \lambda_2, & Q_{21} &= q'_{21} / \lambda_1 \end{aligned} \tag{13}$$

and changing $\partial u_j / \partial x_k$ and $\partial u_\alpha / \partial x_\beta$ in Eqs. (11) and (12) with $\lambda_k \partial u_j / \partial y_k$ and $\lambda_\beta \partial u_\alpha / \partial y_\beta$, respectively, we attain Eq. (3) and expressions for components $\omega_{ij\alpha\beta}$ from Eqs. (11) and (12) after some mathematical calculations. Next we consider the obtaining of the expressions for Q_{11} , ω_{1111} and ω_{1122} given in Eq. (3). From Eqs. (1), (11) and (12) it can be easily attained that

$$q'_{11} = \lambda_1 S'_{11} + S_{11}^0 \frac{\partial u_1}{\partial x_1}, \quad S'_{11} = \lambda_1 \frac{\partial}{\partial \varepsilon_{11}^0} S_{11}^0 \frac{\partial u_1}{\partial x_1} + \lambda_2 \frac{\partial}{\partial \varepsilon_{22}^0} S_{11}^0 \frac{\partial u_2}{\partial x_2}. \tag{14}$$

Taking the relations

$$\lambda_1 \frac{\partial}{\partial \varepsilon_{11}^0} S_{11}^0 = \frac{\lambda_1}{\lambda_1} \frac{\partial S_{11}^0}{\partial \lambda_1} = \frac{1}{\lambda_1} (\lambda + 2\mu) - \frac{1}{(\lambda_1)^2} S_{11}^0, \quad \lambda_2 \frac{\partial}{\partial \varepsilon_{22}^0} S_{11}^0 = \frac{\lambda_2}{\lambda_2} \frac{\partial S_{11}^0}{\partial \lambda_2} = \frac{\lambda}{\lambda_2}, \tag{15}$$

which are obtained from the definition of the parameter λ_i and expression for S_{11}^0 in Eq. (9), and the relations (14) into account, the following mathematical transformations can be made:

$$q'_{11} = (\lambda + 2\mu) \frac{\partial u_1}{\partial x_1} + \lambda \frac{\partial u_2}{\partial x_2} = \lambda_1 (\lambda + 2\mu) \frac{\partial u_1}{\partial y_1} + \lambda \lambda_2 \frac{\partial u_2}{\partial y_2},$$

$$Q_{11} = q'_{11}/\lambda_2 = \frac{\lambda_1}{\lambda_2}(\lambda + 2\mu) \frac{\partial u_1}{\partial y_1} + \lambda \frac{\partial u_2}{\partial y_2} = \omega_{1111} \frac{\partial u_1}{\partial y_1} + \omega_{1122} \frac{\partial u_2}{\partial y_2}.$$

$$\Rightarrow \omega_{1111} = \frac{\lambda_1}{\lambda_2}(\lambda + 2\mu), \quad \omega_{1122} = \lambda. \tag{16}$$

Thus, we obtain the foregoing expressions for components ω_{1111} and ω_{1122} . In this way we obtain the expressions for remain components $\omega_{ij\alpha\beta}$ in Eq. (3) which are differ from zero. These expressions are:

$$\omega_{2211} = \lambda, \quad \omega_{1212} = \omega_{2121} = \frac{2\lambda_2\mu}{\lambda_1 + \lambda_2}, \quad \omega_{1221} = \omega_{2112} = \frac{2(\lambda_2)^2\mu}{\lambda_2(\lambda_1 + \lambda_2)}. \tag{17}$$

Note that the similar discussions of the equations and relations for the circular cylinders in the cylindrical system of coordinates have also been made in a paper by Akbarov (2013b).

Thus with this we restrict ourselves to consideration of the basic equations and relations within the scope of which the motion of the plate-layer is described. In this case the boundary conditions on the upper face plane of the layer can be written as follows.

$$Q_{21}|_{y_2=0} = 0, \quad Q_{22}|_{y_2=0} = -P_0 e^{i\omega t} \delta(y_1). \tag{18}$$

In (18) ω is a frequency of the lineal-located external load with amplitude P_0 , $\delta(y_1)$ is a delta Dirac function.

2.2 Governing field equations for the compressible Newtonian viscous fluid.

Now we consider the field equations of motion of the Newtonian compressible viscous fluid and the density, viscosity constants and pressure related to that we will denote by upper index (1). We use the Euler coordinates in the coordinate system $Oy_1y_2y_3$ which is associated with the initial state of the layer to write these equations. Taking the smallness of the perturbations in the perturbed state in the system under consideration we will identify the Euler and Lagrange coordinates in the coordinate system $Oy_1y_2y_3$. Thus, within the foregoing assumptions, according to Guz (2009), we write field equations for the fluid flow.

The linearized Navier-Stokes equations:

$$\rho_0^{(1)} \frac{\partial v_i}{\partial t} - \mu^{(1)} \frac{\partial v_i}{\partial y_j \partial y_j} + \frac{\partial p^{(1)}}{\partial y_i} - (\lambda^{(1)} + \mu^{(1)}) \frac{\partial^2 v_j}{\partial y_j \partial y_i} = 0, \tag{19}$$

The equation of continuity:

$$\frac{\partial \rho^{(1)}}{\partial t} + \rho_0^{(1)} \frac{\partial v_j}{\partial y_j} = 0, \tag{20}$$

Rheological relations:

$$T_{ij} = \left(-p^{(1)} + \lambda^{(1)} \frac{\partial v_k}{\partial y_k} \right) \delta_{ij} + 2\mu^{(1)} e_{ij}, \tag{21}$$

where

$$e_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial y_j} + \frac{\partial v_j}{\partial y_i} \right). \tag{22}$$

The equation of state:

$$a_0^2 = \frac{\partial p^{(1)}}{\partial \rho^{(1)}}. \tag{23}$$

In Eqs. (19)-(23) $i; j; k = 1, 2, 3$ and the following notation is used: v_i is a component of a perturbation of the velocity vector, $p^{(1)}$ is a perturbation of the pressure, $\mu^{(1)}$ is a coefficient of viscosity, $\lambda^{(1)}$ is the second coefficient of the viscosity, a_0 is a sound speed in the fluid, e_{ij} is a component of a perturbation of the strain rate tensor, T_{ij} is a component of a perturbation of the stress tensor, $\rho^{(1)}$ is a perturbation of the density of the fluid, $\rho_0^{(1)}$ is a density of the fluid in the initial state, i.e. before the perturbation of the fluid and δ_{ij} is a Kronecker symbol. Note that in Eqs. (19)-(21) Einstein summation rule is employed with respect to the repeated indices.

In the present paper we consider the case where

$$v_1 = v_1(y_1, y_2, t), \quad v_2 = v_2(y_1, y_2, t), \quad v_3 = 0. \tag{24}$$

According to Guz (2009), the solution of the system equations (19)-(23) in the case where the relations in (24) take place, is reduced to the finding of two potentials φ and ψ which are determined from the following equations.

$$\left[\left(1 + \frac{\lambda^{(1)} + 2\mu^{(1)}}{a_0^2 \rho_0^{(1)}} \frac{\partial}{\partial t} \right) \Delta - \frac{1}{a_0^2} \frac{\partial^2}{\partial t^2} \right] \varphi = 0, \tag{25}$$

$$\left(\nu^{(1)} \Delta - \frac{\partial}{\partial t} \right) \psi = 0, \quad \Delta = \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial y_2^2},$$

where $\nu^{(1)}$ is a kinematic viscosity, i.e. $\nu^{(1)} = \mu^{(1)} / \rho_0^{(1)}$.

The velocities v_1, v_2 and the pressure $p^{(1)}$ are expressed via the potentials φ and ψ by the following expressions:

$$v_1 = \frac{\partial \varphi}{\partial y_1} + \frac{\partial \psi}{\partial y_2}, \quad v_2 = \frac{\partial \varphi}{\partial y_2} - \frac{\partial \psi}{\partial y_1}, \quad p^{(1)} = \rho_0^{(1)} \left(\frac{\lambda^{(1)} + 2\mu^{(1)}}{\rho_0^{(1)}} \Delta - \frac{\partial}{\partial t} \right) \varphi. \tag{26}$$

Assuming that $p^{(1)} = -(T_{11} + T_{22} + T_{33})/3$, we obtain:

$$\lambda^{(1)} = -\frac{2}{3}\mu^{(1)} \quad (27)$$

This completes the field equation of the fluid flow which is considered in the present paper. As fluid occupies the half-plane $(-\infty < y_1 < +\infty, -\infty < y_2 < -\lambda_2 h)$, we assume that

$$|v_i| < const, \quad \left| \frac{\partial v_i}{\partial y_j} \right| < const, \quad i, j = 1, 2 \text{ as } y_2 \rightarrow -\infty, \quad (28)$$

and there is not reflected waves from $y_2 = -\infty$.

2.3 Contact conditions on the interface between the fluid and pre-strained layer

We assume that the velocities and forces acting on the interface between the fluid and layer are continuous. In other words, we assume that

$$\begin{aligned} \frac{\partial u_1}{\partial t} \Big|_{y_2=-\lambda_2 h} &= v_1 \Big|_{y_2=-\lambda_2 h}, & \frac{\partial u_2}{\partial t} \Big|_{y_2=-\lambda_2 h} &= v_2 \Big|_{y_2=-\lambda_2 h}, \\ Q_{21} \Big|_{y_2=-\lambda_2 h} &= T_{21} \Big|_{y_2=-\lambda_2 h}, & Q_{22} \Big|_{y_2=-\lambda_2 h} &= T_{22} \Big|_{y_2=-\lambda_2 h}. \end{aligned} \quad (29)$$

This completes the formulation of the problem. It should be noted that, with corresponding obvious changes, the foregoing problem formulation can be remake for the case where the fluid is inviscid. Moreover, in the case where $\lambda_1 = \lambda_2 = 1.0$ the foregoing formulation relates to the corresponding classical problem of hydro-elastodynamics.

3 Method of solution

Below, we use the dimensionless coordinates $\bar{y}_k = y_k/h$ and omit the over bar on the coordinates. According to the boundary condition (18), we represent the sought values as

$$g(y_1, y_2, t) = \bar{g}(y_1, y_2)e^{i\omega t}. \quad (30)$$

Substituting the expression (30) into the foregoing equations and relations, and replacing the derivatives $\partial(\bullet)/\partial t$ and $\partial^2(\bullet)/\partial t^2$ with $i\omega(\bullet)$ and $-\omega^2(\bullet)$ respectively we obtain corresponding equations, boundary and contact conditions for the amplitudes of the sought values. For solution to these equations we use the exponential Fourier transformation with respect to the y_1 coordinate:

$$f_F(s, y_2) = \int_{-\infty}^{+\infty} f(y_1, y_2)e^{-isy_1} dy_1. \quad (31)$$

Taking the problem symmetry with respect to $y_1 = 0$ into account, we can represent the originals of the sought values as follows.

$$\begin{aligned}
 u_1 &= \frac{1}{\pi} \int_0^{\infty} u_{1F}(s, y_2) \sin(sy_1), & u_2 &= \frac{1}{\pi} \int_0^{\infty} u_{2F}(s, y_2) \cos(sy_1), \\
 Q_{11} &= \frac{1}{\pi} \int_0^{\infty} Q_{11F}(s, y_2) \cos(sy_1), & Q_{22} &= \frac{1}{\pi} \int_0^{\infty} Q_{22F}(s, y_2) \cos(sy_1), \\
 Q_{12} &= \frac{1}{\pi} \int_0^{\infty} Q_{12F}(s, y_2) \sin(sy_1), & Q_{21} &= \frac{1}{\pi} \int_0^{\infty} Q_{21F}(s, y_2) \sin(sy_1), \\
 \varphi &= \frac{1}{\pi} \int_0^{\infty} \varphi_F(s, y_2) \cos(sy_1), & \psi &= \frac{1}{\pi} \int_0^{\infty} \psi_F(s, y_2) \sin(sy_1), \\
 v_1 &= \frac{1}{\pi} \int_0^{\infty} v_{1F}(s, y_2) \sin(sy_1), & v_2 &= \frac{1}{\pi} \int_0^{\infty} v_{2F}(s, y_2) \cos(sy_1), \\
 T_{11} &= \frac{1}{\pi} \int_0^{\infty} T_{11F}(s, y_2) \cos(sy_1), & T_{22} &= \frac{1}{\pi} \int_0^{\infty} T_{22F}(s, y_2) \cos(sy_1), \\
 T_{12} &= \frac{1}{\pi} \int_0^{\infty} T_{12F}(s, y_2) \sin(sy_1), & T_{21} &= \frac{1}{\pi} \int_0^{\infty} T_{21F}(s, y_2) \sin(sy_1).
 \end{aligned} \tag{32}$$

First, we consider the solution of the equations related to the Fourier transformation of the quantities related to the plate-layer, i.e. to the solution of the equations which are obtained from the equations (2), (3), (16) and (17) by employing Fourier transformation (32). Thus, substituting the expressions (32) into the equations (2) and (3), and doing some mathematical manipulations we obtain the following equations for the u_{1F} and u_{2F} .

$$Au_{1F} - B \frac{du_{2F}}{dy_2} + C \frac{d^2u_{1F}}{dy_2^2} = 0, \quad Du_{2F} + B \frac{du_{1F}}{dy_2} + G \frac{d^2u_{2F}}{dy_2^2} = 0, \tag{33}$$

where

$$\begin{aligned}
 A &= X^2 - s^2 \omega_{1111}, & B &= s(\omega_{1122} + \omega_{2121}), & C &= \omega_{2112}, \\
 D &= X^2 - s^2 \omega_{1221}, & G &= \omega_{2222}, & X^2 &= \omega^2 h^2 / c_2^2, & c_2 &= \sqrt{\mu / \rho}.
 \end{aligned} \tag{34}$$

Introducing the notation

$$A_0 = \frac{AG + B^2 + CD}{CG}, \quad B_0 = \frac{BD}{CG},$$

$$k_1 = \sqrt{-\frac{A_0}{2} + \sqrt{\frac{A_0^2}{4} - B_0}}, \quad k_2 = \sqrt{-\frac{A_0}{2} - \sqrt{\frac{A_0^2}{4} - B_0}}, \quad (35)$$

we can write the solution of the equation (33) as follows.

$$u_{2F} = Z_1 e^{k_1 y_2} + Z_2 e^{-k_1 y_2} + Z_3 e^{k_2 y_2} + Z_4 e^{-k_2 y_2},$$

$$u_{1F} = Z_1 a_1 e^{k_1 y_2} + Z_2 a_2 e^{-k_1 y_2} + Z_3 a_3 e^{k_2 y_2} + Z_4 a_4 e^{-k_2 y_2}, \quad (36)$$

where

$$a_1 = \frac{-D - Gk_1^2}{Bk_1^2}, \quad a_2 = -a_1, \quad a_3 = \frac{-D - Gk_2^2}{Bk_2^2}, \quad a_4 = -a_3. \quad (37)$$

Using the equations (3) and (36) we also write expressions for the Fourier transformations Q_{21F} and Q_{22F} of the corresponding stresses which enter the boundary condition (18) and contact condition (29).

$$Q_{21F} = Z_1 (\omega_{2112} k_1 a_1 - s \omega_{2121}) e^{k_1 y_2} + Z_2 (-\omega_{2112} k_1 a_2 - s \omega_{2121}) e^{-k_1 y_2} +$$

$$Z_3 (\omega_{2112} k_2 a_3 - s \omega_{2121}) e^{k_2 y_2} + Z_4 (-\omega_{2112} k_2 a_3 - s \omega_{2121}) e^{-k_2 y_2},$$

$$Q_{22F} = Z_1 (s \omega_{2211} a_1 + k_1 \omega_{2222}) e^{k_1 y_2} + Z_2 (s \omega_{2211} a_2 - k_1 \omega_{2222}) e^{-k_1 y_2} +$$

$$Z_3 (s \omega_{2211} a_3 + k_2 \omega_{2222}) e^{k_2 y_2} + Z_4 (s \omega_{2211} a_4 - k_2 \omega_{2222}) e^{-k_2 y_2}. \quad (38)$$

This completes the consideration of the determination of the Fourier transformation of the values related to the plate-layer. Now we consider the determination of the Fourier transformations of the quantities related to the fluid flow. First, we consider the determination of the φ_F and ψ_F from the Fourier transformation of the equations in (25), which taking the relation (27) into account and the relation

$$\varphi_F = \omega h^2 \tilde{\varphi}_F, \quad \psi_F = \omega h^2 \tilde{\psi}_F \quad (39)$$

can be written as follows

$$\frac{d^2 \tilde{\varphi}_F}{dy_2^2} + \left(\frac{\Omega_1^2}{1 + i4\Omega_1^2 / (3N_w^2)} - s^2 \right) \tilde{\varphi}_F = 0, \quad \frac{d^2 \tilde{\psi}_F}{dy_2^2} - (s^2 + iN_w^2) \tilde{\psi}_F = 0, \quad (40)$$

where

$$\Omega_1 = \frac{\omega h}{a_0}, \quad N_w^2 = \frac{\omega h^2}{\nu^{(1)}}. \tag{41}$$

The dimensionless number N_w in (41) can be taken as Womersley number and characterizes the influence of the fluid viscosity on the mechanical behavior of the system under consideration. When the Womersley number is large (around 10 or greater), it shows that the flow is dominated by oscillatory inertial forces. When the Womersley number is low, viscous forces tend to dominate the flow. However, for hydro-elastodynamic problems the mentioned “large” and “low” limits for the Womersley number can change significantly.

The dimensionless frequency Ω_1 in (41) can be taken as the parameter which characterizes the compressibility of the fluid on the mechanical behavior of the system under consideration. Thus, taking the conditions (28) into consideration, the solutions to the equations in (40) are found as follows

$$\tilde{\varphi}_F = Z_5 e^{\delta_1 y_2}, \quad \tilde{\psi}_F = Z_6 e^{\gamma_1 y_2}, \tag{42}$$

where

$$\delta_1 = \sqrt{s^2 - \frac{\Omega_1^2}{1 + i4\Omega_1^2/(3N_w^2)}}, \quad \gamma_1 = \sqrt{s^2 + iN_w^2}. \tag{43}$$

Using (42) and (39) we obtain the following expressions for the velocities, pressure and stresses of the fluid from the Fourier transformations of the equations (21), (22) and (26).

$$\begin{aligned} v_{1F} &= \omega h \left[-Z_5 s e^{\delta_1 y_2} + Z_6 e^{\gamma_1 y_2} \right], \quad v_{2F} = \omega h \left[Z_5 \delta_1 e^{\delta_1 y_2} - Z_6 s e^{\gamma_1 y_2} \right], \\ T_{22F} &= \mu^{(1)} \omega \left[Z_5 \left(\frac{4}{3} \delta_1^2 + \frac{2}{3} s^2 - R_0 \right) e^{\delta_1 y_2} + Z_6 \left(-s \gamma_1 - \frac{2}{3} s \gamma_1 \right) e^{\gamma_1 y_2} \right], \\ T_{21F} &= -\mu^{(1)} \omega \left[2s \delta_1 Z_5 e^{\delta_1 y_2} + (s^2 + \gamma_1^2) Z_6 e^{\gamma_1 y_2} \right], \quad p_F^{(1)} = \mu^{(1)} \omega R_0 Z_5 e^{\delta_1 y_2}, \end{aligned} \tag{44}$$

where

$$R_0 = -\frac{4}{3} \frac{\Omega_1^2}{1 + i4\Omega_1^2/(3N_w^2)} - N_w^2. \tag{45}$$

Substituting expressions (36), (38) and (44) into the boundary condition (18) and contact condition (29) we obtain system of equations with respect to the unknowns

Z_1, Z_2, \dots, Z_6 through which the sought values are determined. The mentioned equation can be expressed as follows.

$$\begin{aligned}
 (Q_{21}/\mu)|_{y_2=0} &= Z_1\alpha_{11} + Z_2\alpha_{12} + Z_3\alpha_{13} + Z_4\alpha_{14} = 0, \\
 (Q_{22}/\mu)|_{y_2=0} &= Z_1\alpha_{21} + Z_2\alpha_{22} + Z_3\alpha_{23} + Z_4\alpha_{24} = -P_0/\mu, \\
 \frac{\partial u_{1F}}{\partial t} \Big|_{y_2=-\lambda_2 h} - v_{1F}|_{y_2=-\lambda_2 h} & \\
 = i\omega(Z_1\alpha_{31} + Z_2\alpha_{32} + Z_3\alpha_{33} + Z_4\alpha_{34}) - \omega h(Z_5\alpha_{35} + Z_6\alpha_{36}) &= 0, \\
 \frac{\partial u_{2F}}{\partial t} \Big|_{y_2=-\lambda_2 h} - v_{2F}|_{y_2=-\lambda_2 h} & \\
 = i\omega(Z_1\alpha_{41} + Z_2\alpha_{42} + Z_3\alpha_{43} + Z_4\alpha_{44}) - \omega h(Z_5\alpha_{45} + Z_6\alpha_{46}) &= 0, \\
 (Q_{21}/\mu)|_{y_2=-\lambda_2 h} - (T_{21}/\mu)|_{y_2=-\lambda_2 h} & \\
 = Z_1\alpha_{51} + Z_2\alpha_{52} + Z_3\alpha_{53} + Z_4\alpha_{54} - M(Z_5\alpha_{55} + Z_6\alpha_{56}) &= 0, \\
 (Q_{22}/\mu)|_{y_2=-\lambda_2 h} - (T_{22}/\mu)|_{y_2=-\lambda_2 h} & \\
 = Z_1\alpha_{61} + Z_2\alpha_{62} + Z_3\alpha_{63} + Z_4\alpha_{64} - M(Z_5\alpha_{65} + Z_6\alpha_{66}) &= 0,
 \end{aligned} \tag{46}$$

where

$$M = \frac{\mu^{(1)}\omega}{\mu}. \tag{47}$$

The expressions of the coefficients $\alpha_{nm}(n; m = 1, 2, \dots, 6)$ can be easily determined from expressions (36), (38) and (44), and therefore we do not give here these expressions. Thus, unknowns Z_1, Z_2, \dots, Z_6 in the equations (46) can be determined via the formulae.

$$Z_k = \frac{\det \|\beta_{nm}^k\|}{\det \|\alpha_{nm}\|}, \tag{48}$$

Note the matrix (β_{nm}^k) is obtained from the matrix (α_{nm}) by the replacing of the $k - th$ column of the (α_{nm}) by the column $(0, -P_0/\mu, 0, 0, 0, 0)^T$.

Now we consider the calculation of the integrals in (32). For this purpose first we consider the following reasoning. If we take the Fourier transformation parameter s as the wavenumber then the equation

$$\det \|\alpha_{nm}\| = 0, \quad n; m = 1, 2, \dots, 6, \tag{49}$$

coincides with the dispersion equation of the waves propagated in the direction of the Oy_1 axis in the system under consideration. It should be noted that, according to the well-known physic-mechanical considerations, the equation (49) must have complex roots only. This character of the roots is caused with the viscosity of the fluid. However, as usual, the viscosity of the Newtonian fluids is insignificant in the qualitative sense and therefore in many cases within the scope of the PC calculation accuracy the equation (49) has real roots. Consequently these roots become singular points of the integrated expressions in the integrals (32). Therefore, according to works by Lamb (1904), Tsang (1978), Jensen et al (2011) and many others listed in these references, we will evaluate the wavenumber integrals (32) along the Sommerfeld contour (Fig. 1b) in the complex plane $s = s_1 + is_2$ and in this way the real roots of Eq. (49) are avoided.

Thus, using the presentation (30), we can determine the sought values through the following two type relations.

$$\{Q_{22}, Q_{11}, u_2, T_{22}, T_{11}, v_2\} = \frac{1}{\pi} Re \left\{ e^{i\omega t} \int_C [Q_{22F}, Q_{11F}, u_{2F}, T_{22F}, T_{11F}, v_{2F}] \cos(sy_1) ds \right\},$$

$$\{Q_{21}, Q_{12}, u_1, T_{21}, v_1\} = \frac{1}{\pi} Re \left\{ e^{i\omega t} \int_C [Q_{21}, Q_{12F}, u_{1F}, T_{21F}, v_{1F}] \sin(sy_1) ds \right\}. \tag{50}$$

According to Fig. 1b, we can write the following relation.

$$\int_C f(s) \cos(sy_1) ds = i \int_0^{s_2^*} f(is_2) \cos(is_2y_1) ds_2 + \int_0^\infty f(s_1 + is_2^*) \cos((s_1 + is_2^*)y_1) ds_1,$$

$$\int_C f(s) \sin(sy_1) ds = i \int_0^{s_2^*} f(is_2) \sin(is_2y_1) ds_2 + \int_0^\infty f(s_1 + is_2^*) \sin((s_1 + is_2^*)y_1) ds_1. \tag{51}$$

Taking the fact that the values of the integrals $\int_C f(s) \cos(sy_1) ds$ and $\int_C f(s) \sin(sy_1) ds$ are independent on the values of the parameter s_2^* into account, as usual (see, for example Jensen et al (2011) and Tsang (1978)), to simplify the

calculation of these integrals, the parameter s_2^* is assumed as a small parameter. According to this assumption and to the relation

$$\left| \int_0^{s_2^*} f(is_2) \cos(is_2 y_1) ds_2 \right| = O(s_2^*), \quad \left| \int_0^{s_2^*} f(is_2) \sin(is_2 y_1) ds_2 \right| = O(s_2^*),$$

we use the following approximate expressions for calculation of the foregoing integrals

$$\int_C f(s) \cos(s y_1) ds \approx \int_0^\infty f(s_1 + is_2^*) \cos((s_1 + is_2^*) y_1) ds_1,$$

$$\int_C f(s) \sin(s y_1) ds \approx \int_0^\infty f(s_1 + is_2^*) \sin((s_1 + is_2^*) y_1) ds_1. \tag{52}$$

The accuracy of expressions in (52) with respect to values of the parameter s_2^* was discussed in a paper by Akbarov and Ilhan (2013).

Moreover under calculation procedure, the improper integrals $\int_0^{+\infty} f(s_1) \cos(s_1 y_1) ds_1$ and $\int_0^{+\infty} f(s_1) \sin(s_1 y_1) ds_1$ in (52) are replaced by the corresponding definite integrals $\int_0^{+S_1^*} f(s_1) \cos(s_1 y_1) ds_1$ and $\int_0^{+S_1^*} f(s_1) \sin(s_1 y_1) ds_1$ respectively. The values of S_1^* are determined from the convergence requirement of the numerical results. Note that under calculation of the latter integrals, the integration intervals are further divided into a certain number of shorter intervals, which are used in the Gauss integration algorithm. In this integration procedure it is assumed that in each of the shorter intervals the sampling intervals of the numerical integration Δs_1 must satisfy the relation $|\Delta s_1| \ll \min \{s_2^*, 1/y_1\}$. All these procedures are performed automatically with the PC by use of the corresponding programs constructed by the authors in MATLAB.

This completes the discussions related to the algorithms employing for calculation of the wave-number integrals in the form (32). Note that after some obvious changing the foregoing solution method can be applied also for the case where the fluid is inviscid.

4 Numerical results and discussions

It follows from the foregoing discussions that the problem under consideration is characterized through the dimensionless parameters Ω_1 and N_w which are deter-

mined by the expressions in (41), M which is determined with the expression (47), λ/μ where λ and μ are the mechanical constants which enter the expression of the elastic potential (7), and λ_1 through which the initial strains in the layer are characterized. Note that the case where $\Omega_1 = 0$ corresponds to the incompressible fluid, but the case where $1/N_w = 0$ to the inviscid fluid.

Under numerical investigation we assume that the values of the mechanical constants and the density of the plate material are $\mu = 1.86 \times 10^9 Pa$, $\lambda = 3.96 \times 10^9 Pa$ and $\rho = 1160 kg/m^3$, but the material of the fluid is Glycerin with viscosity coefficient $\mu^{(1)} = 1,393 kg/(m \cdot s)$, density $\rho = 1260 kg/m^3$ and sound speed $a_0 = 1459.5 m/s$ [Guz (2009)]. We introduce also the notation $c_2 = \sqrt{\mu/\rho}$ which is the shear wave propagation velocity in the layer material in the case where the initial strains are absent in that. Note that the values selected above for the constants λ , μ and ρ , and related to the plate material under absent of the initial strains corresponds to the Plexiglass (or Lucite) [see Guz (2004); Lai-Yu et al (2006)]. Consequently, these values have real meaning and numerical results attained in the case where the initial strains are absent in the plate, i.e. in the case where $\lambda_1 = \lambda_2 = 1.0$ in the linearized elastic relations (3), (16) and (17), can be regarded the hydro-elastic system consisting of the plate made of Plexiglass and half-plane filled by the Glycerin.

Thus, after the selection the values of materials constants the foregoing dimensionless parameters can be determined through the three quantities: h (the thickness of the plate-layer), ω (the frequency of the time-harmonic external forces) and λ_1 (the elongation parameter through which the initial strains in the layer are determined). In the present paper we will consider namely the influence of these three parameters on the distribution of the velocities and stresses on the interface plane between the pre-strained plate-layer and fluid.

With respect to the plate-layer thickness we consider two cases: the first case we call the "thin plate case" for which $0.001 m \leq h \leq 0.005 m$, but the second we call the "thick plate case" for which $0.05 m \leq h \leq 1.0 m$. For the "thin plate case" we assume that $5 Hz < \omega \leq 300 Hz$, but for the "thick plate case" $5 Hz < \omega \leq 1000 Hz$.

Before consideration of the numerical results we note the following reasoning. According to the mechanical consideration and a lot of numerical results (which are not given here), it can be predicted that the influence of the fluid viscosity on its motion in the case under consideration must be notable, namely in the "thin plate case" for low frequency of the forced vibration. But the influence of the fluid compressibility can be neglected in the "thin plate case" for the frequencies $5 Hz < \omega \leq 300 Hz$, in other words in these cases the results obtained for the compressible and incompressible fluid models coincides with each other with very high accuracy. Also, according to the mechanical consideration, it can be predicted that

the influence of the fluid compressibility on its motion must be considerable in the "thick plate case" under relatively high frequency of the forced vibration. Based on this reasoning we select the foregoing change range of the frequency for "thin" and "thick plate cases" and under obtaining numerical results related to the "thin plate case" we will assume that the fluid is compressible one.

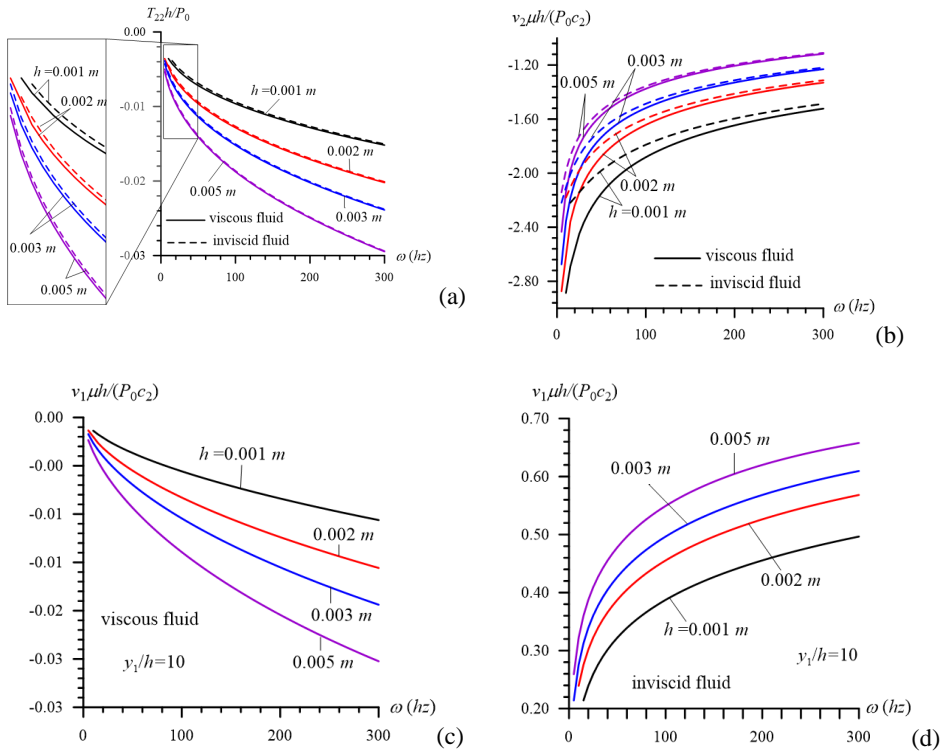


Figure 2: The frequency response of T_{22} (a), v_2 (b), v_1 (c, for a viscous fluid) and also v_1 (d, for an inviscid fluid) in the “thin plate case” under absent of the initial strains in the plate.

First, we consider the case where $\omega t = 2\pi n$ ($n = 0, 1, 2, \dots$), i.e. the case where $\cos(\omega t) = 1$ and $\sin(\omega t) = 1$ in Eq. (50). Thus, we analyze numerical results obtained for the T_{22} , v_2 and v_1 , and calculated on the interface plane between the layer and fluid. Consider graphs given in Fig. 2 which illustrate the frequency response of the T_{22} (Fig. 2a), v_2 (Fig. 2b), v_1 (Fig. 2c for a viscous fluid) and also v_1 (Fig. 2d for an inviscid fluid). Under obtaining these results the values related to the T_{22} and v_2 are calculated at $y_1/h = 0$, but the values related to the

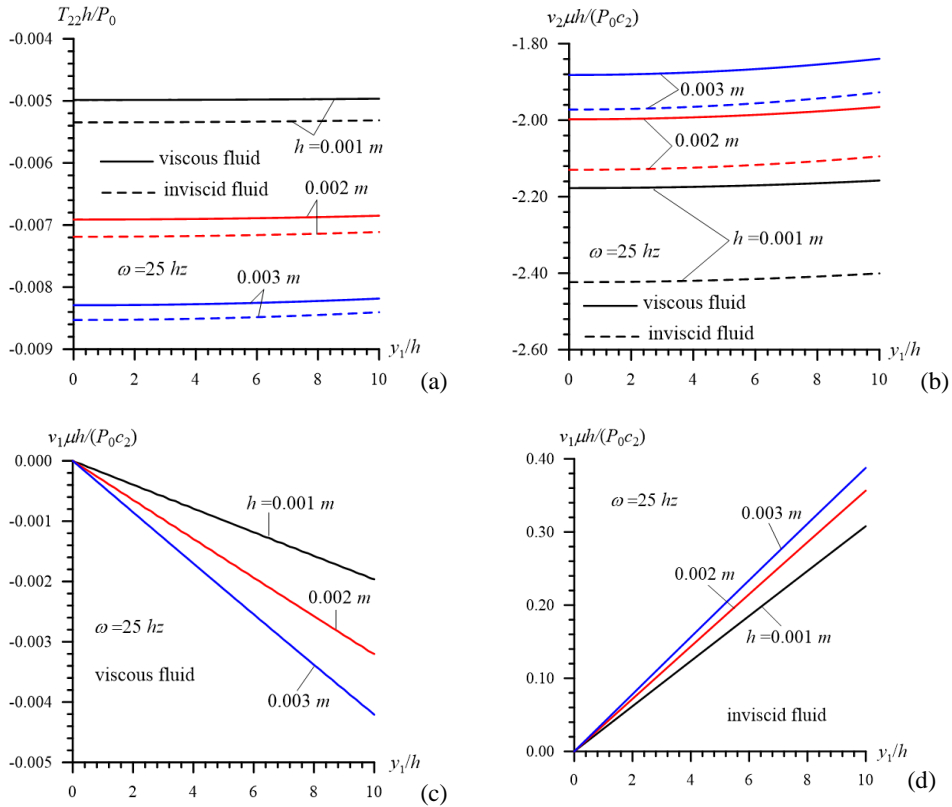


Figure 3: The distribution of T_{22} (a), v_2 (b), v_1 (c, for a viscous fluid) and also v_1 (d, for an inviscid fluid) with respect to the y_1/h in the “thin plate case” under absent of the initial strains.

v_1 at $y_1/h = 10$ and it is assumed that the initial strains in the plate are absent, i.e. it is assumed that $\lambda_1 = 1.0$. It follows from these results that as a result of the fluid viscosity the absolute values of the stress T_{22} and velocity v_2 decrease. The influence of the fluid viscosity on the velocity v_2 is more significant than on the stress T_{22} and the magnitude of the mentioned influence decrease with frequency ω .

Note that this conclusion agrees with the well-known mechanical considerations related to the influence of the viscosity of the systems on their vibration.

It should be especially noted - the more considerable influence of the fluid viscosity on the frequency response of the velocity v_1 . This considerable influence is explained by the fact that for the inviscid fluid, the contact conditions

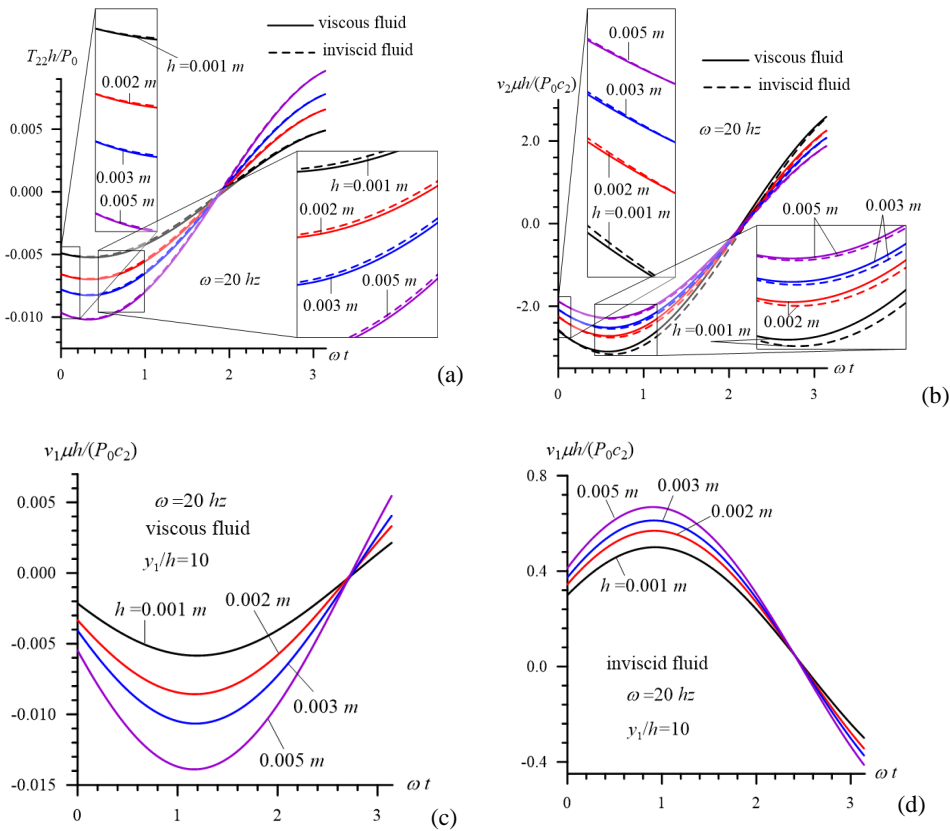


Figure 4: Graphs of the dependencies among T_{22} (a), v_2 (b), v_1 (c, for a viscous fluid) and also v_1 (d, for an inviscid fluid) and ωt in the “thin plate case” under absent of the initial strains in the plate.

$\partial u_2/\partial t|_{y_2=-\lambda_2 h} = v_2|_{y_2=-\lambda_2 h}$ and $Q_{21}|_{y_2=-\lambda_2 h} = T_{21}|_{y_2=-\lambda_2 h}$ in (29), disappear. As a result of this disappearance, the sign of the velocity v_1 obtained for the viscous fluid, is opposite that obtained for the inviscid fluid. At the same time, the results given in Figs. 2c and 2d show that the absolute values of the velocity v_1 obtained for the viscous fluid are less significant than the corresponding values of this velocity obtained for the inviscid fluid. Consequently, the foregoing results show that in the case under consideration, as well as in other similar cases related to plate-fluid interaction problems, under determination of the fluid flow velocity v_1 on the interface plane and in the near vicinity of this plane, the viscous fluid model must be used.

At the same time, it follows from the results given in Fig. 2 that in the considered

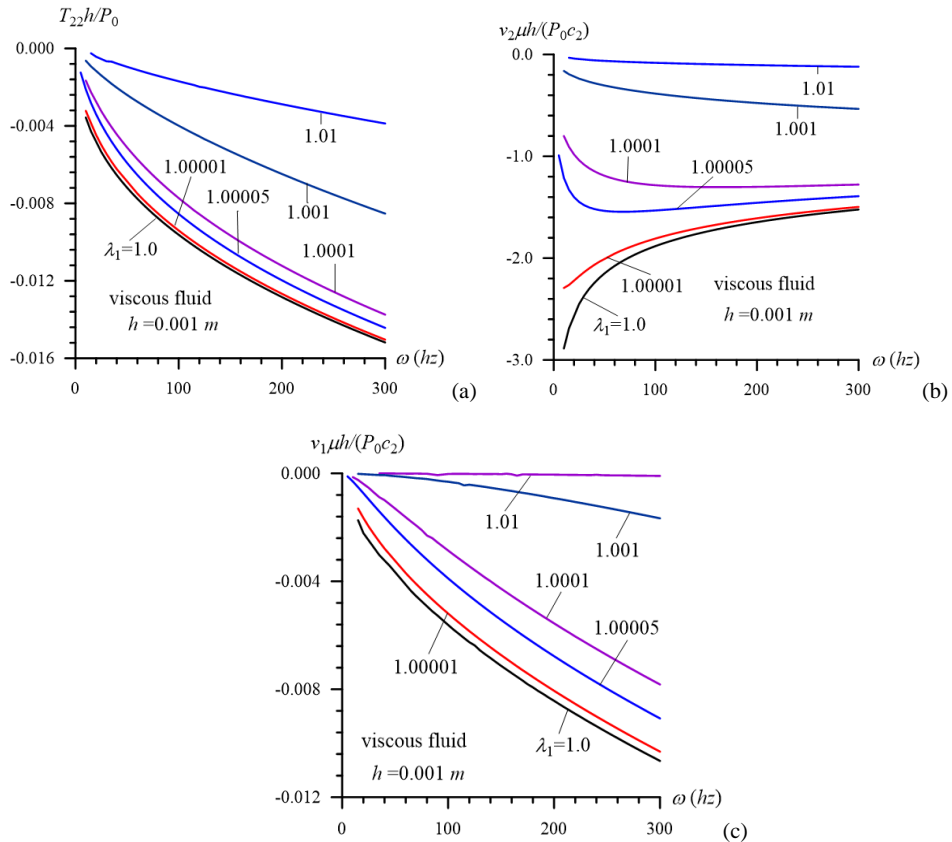


Figure 5: The influence of the initial stretching of the plate on the frequency response of T_{22} (a), v_2 (b) and v_1 (c, for a viscous fluid) in the “thin plate case” under $h = 0.001$ m.

change range for the frequency the absolute values of the T_{22} and v_1 increase, but the values of the v_2 decrease with the frequency ω .

Consider the distribution of the considered quantities with respect to the dimensionless coordinate y_1/h the graphs of which are given in Fig. 3a (for T_{22}), Fig. 3b (for v_2), Fig. 3c (for v_1 in the viscous fluid case) and Fig. 3d (also for v_1 in the inviscid fluid case) and are constructed under $\lambda_1 = 1.0$. It follows from the results given in Fig. 3, as can be predicted, the absolute maximum values for the stress T_{22} and the velocity v_2 are obtained at $y_1/h = 0$. According to the problem symmetry with respect to the plane $y_1 = 0$, the velocity v_1 is equal to zero at $y_1/h = 0$.

Consequently, the absolute maximum values of the velocity v_1 are obtained at far

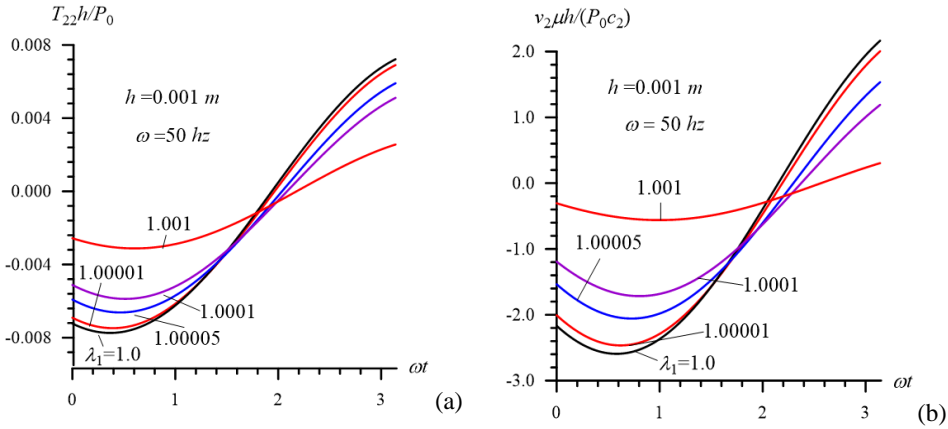


Figure 6: The influence of the initial stretching of the layer (i.e. of the parameter λ_1) on the dependencies among $T_{22}h/P_0$ (a), $v_2\mu h/(P_0c_2)$ (b) and ωt .

from the point $y_1/h = 0$ and in the considered change range of the dimensionless coordinate y_1/h these points are not reached in Figs. 3c and 3d.

According to the mechanical consideration the values of the T_{22} , v_2 and v_1 must decay as $y_1/h \rightarrow \infty$. This decay is observed for the stress T_{22} and velocity v_2 from the Figs. 3a and 3b, although this decay is very weak in the “thin plate case” and the magnitude of the decay increases with the plate thickness. Therefore the considerable illustration of the decay of the studied quantities with respect to the dimensionless coordinate y_1/h will be clearly illustrated under consideration the “thick plate case”.

We recall that the foregoing results are obtained from the expressions in (50) in the cases where $\omega t = 2n\pi$ ($n = 0, 1, 2, \dots$). Now we consider numerical results related to the dependencies among $T_{22}h/P_0$, $v_2\mu h/(P_0c_2)$, $v_1\mu h/(P_0c_2)$ and ωt in the case where $0 \leq \omega t \leq \pi$. Graphs of these dependencies in the case where $\lambda_1 = 1.0$ are given in Figs. 4a (for T_{22}), 4b (for v_2), 4c (for v_1 in the viscous fluid case) and 4d (for v_1 in the inviscid fluid case). It follows from these graphs that the absolute maximum values of the studied quantities arise in the cases where $\omega t \neq 0 + n\pi$ ($n = 0, 1, 2, \dots$). In other words, the absolute maximum values of the studied quantities arise at $\omega t = (\omega t)_* + n\pi$ and the values of $(\omega t)_*$ can be easily determined from Fig. 4 for each considered case. However, the absolute maximum values of the external loading arise, namely at $\omega t = 0 + n\pi$. This means phase shifting of the studied quantities with respect to the external loading. It follows from Fig. 4 that this phase shifting is more considerable for the velocities v_2 and v_1 .

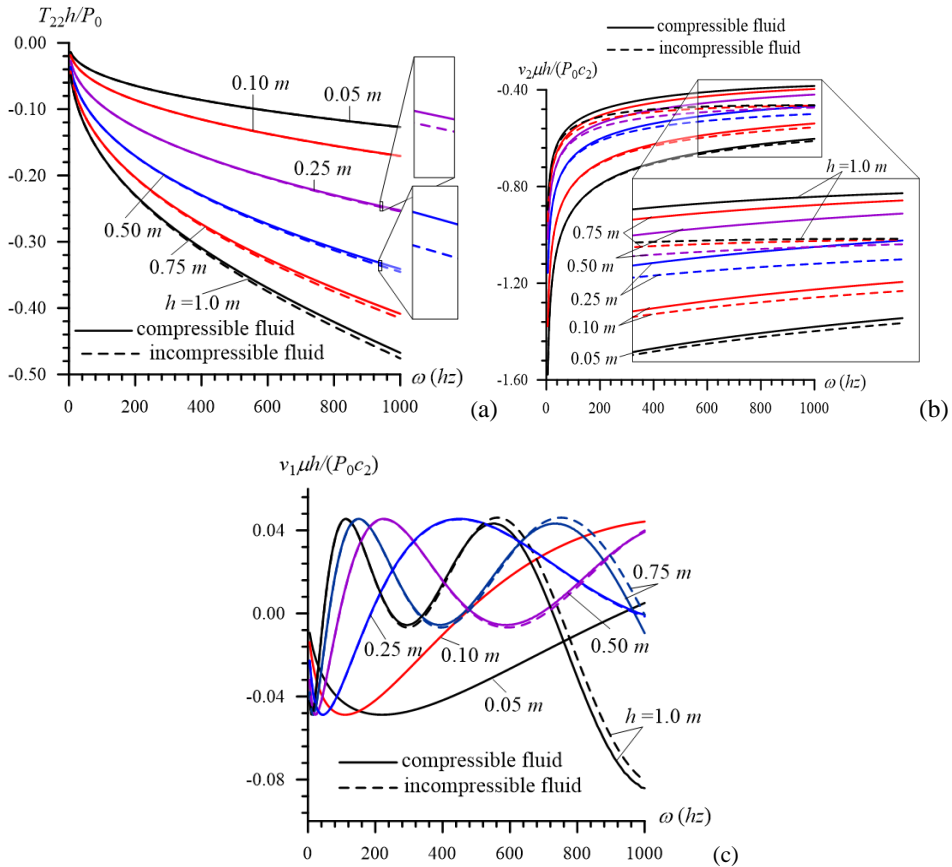


Figure 7: The frequency response of T_{22} (a), v_2 (b) and v_1 (c, for a viscous fluid) in the “thick plate case” under absent of the initial strains in the plate.

All the numerical results discussed above have been obtained in the case where the initial strains in the plate layer are absent. Consequently, the foregoing results can be related for the case where the plate material is a Plexiglass (Lucite) and the fluid is Glycerin. In other words, the foregoing results have real application field for the noted constituents of the system under consideration. Now we consider the results illustrated the influence of the initial strains of the plate on the frequency response of the studied quantities. For this purpose, we consider the graphs shown in Fig. 5 which indicate the influence of the parameter λ_1 on the frequency response of the T_{22} (Fig. 5a), v_2 (Fig. 5b) and v_1 (Fig. 5c). It follows from Fig. 5 that the initial stretching of the plate-layer causes to decrease the absolute values of the

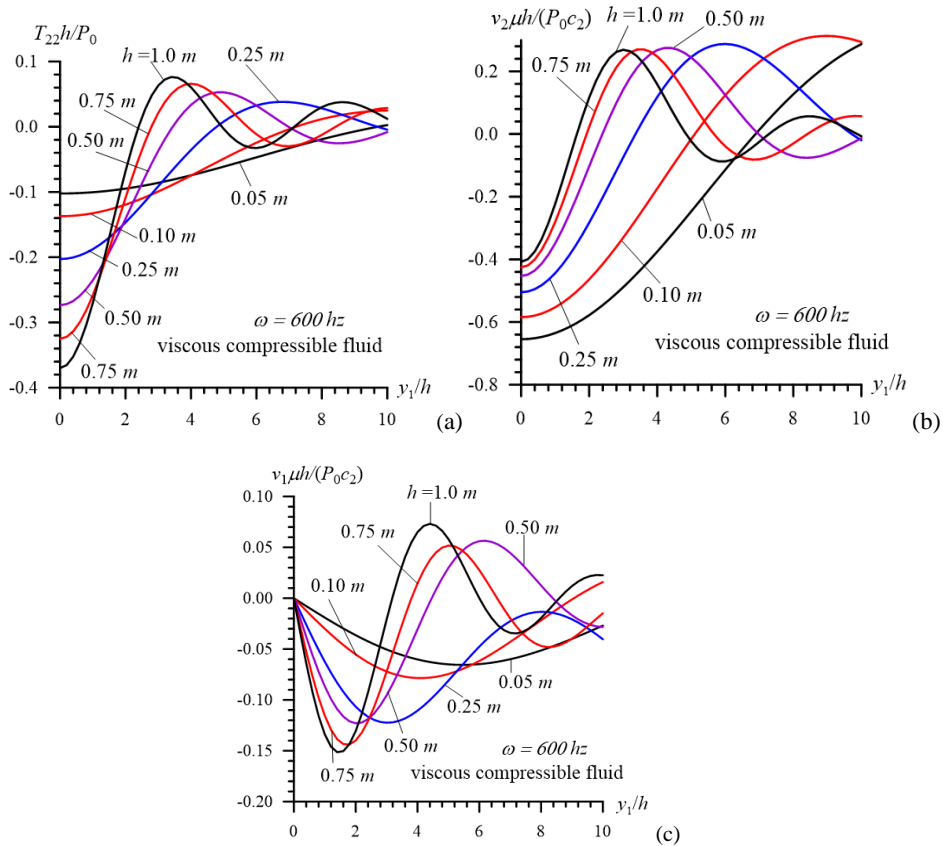


Figure 8: The distribution of T_{22} (a), v_2 (b), v_1 (c, for a viscous fluid) and also v_1 (d, for an inviscid fluid) with respect to the y_1/h in the “thick plate case” under absent of the initial strains in the plate and under $\omega = 600 \text{ Hz}$.

studied quantities significantly. Moreover, it follows from the Fig. 5b that the initial stretching of the plate-layer effects also on the character of the frequency response of the velocity v_2 , i.e. for the relatively small values of the parameter λ_1 (for instance in the cases where $\lambda_1 \leq 1.00001$) the absolute values of the v_2 increase, but for the relatively great values of the parameter λ_1 (for instance in the cases where $\lambda_1 \leq 1.00005$) the absolute values of the v_2 decrease monotonically with the frequency ω .

Also, we consider the influence of the initial stretching of the plate-layer on the dependence among $T_{22}h/P_0$ (Fig. 6a), $v_2\mu h/(P_0c_2)$ (Fig. 6b) and ωt . It follows from Fig. 6 that the initial stretching of the plate-layer causes to increase of the

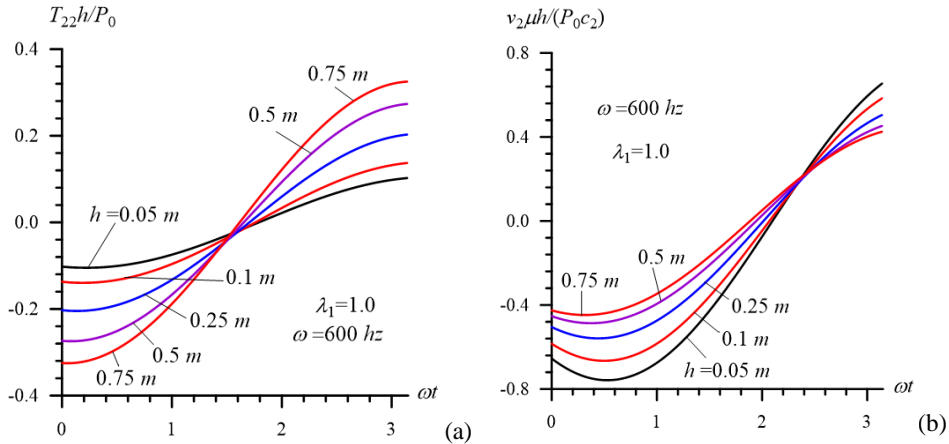


Figure 9: Graphs of the dependencies among T_{22} (a), v_2 (b) and ωt in the “thick plate case” under absent of the initial strains in the plate and under $\omega = 600 \text{ hz}$.

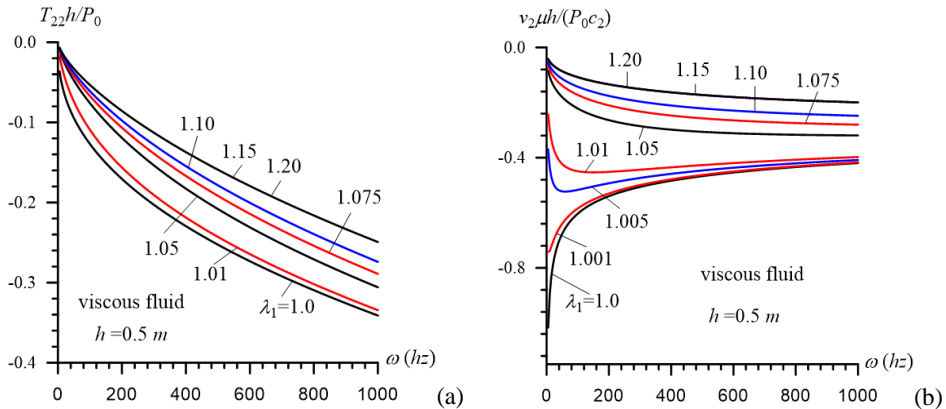


Figure 10: The influence of the initial stretching of the plate on the frequency response of T_{22} (a) and v_2 (b) in the “thick plate case” under $h = 0.5 \text{ m}$.

values of the phase shifting $(\omega t)_*$.

Thus, with the above, we restrict ourselves to consideration of the numerical results related to the “thin plate case”. Now we consider numerical results related to the “thick plate case”, according to which, we assume that $0.05 \text{ m} \leq h \leq 1.0 \text{ m}$. First, as above, we assume that $\omega t = 2n\pi$ ($n = 0, 1, 2, \dots$) and analyze the graphs given in Fig. 7 which illustrate the frequency response of T_{22} (Fig. 7a), v_2 (Fig. 7b) and v_1 (Fig. 7c) under $\lambda_1 = 1.0$. Note that under construction these graphs we

assume that the fluid is viscous one and the results obtained for the T_{22} and v_2 in the inviscid fluid case coincide with the corresponding ones shown in Fig. 7. However, the results obtained for the velocity v_1 obtained in the inviscid fluid case, as in the “thin plate case” differ significantly from corresponding ones obtained in the viscous fluid case. Taking these discussions into account below, as in Fig. 7, we will analyze numerical results related to the viscous fluid case. Thus, we turn again to the results given in Fig. 7 and note that these results are given in the compressible and incompressible fluid cases simultaneously. It follows from the analyses of the graphs illustrated in Fig. 7 that the fluid incompressibility causes to increase of the absolute values of the stress T_{22} and of the velocity v_2 and this influence is more considerable for the velocity v_2 . The magnitude of the mentioned influence increase monotonically with the thickness of the plate h and with the frequency of the external forced vibration ω , in other words with increasing of the values of the parameter Ω_1 in (41). According to the numerical results obtained for various values of the parameter Ω_1 , in general, it can be noted that the influence of the compressibility of the fluid on the foregoing results becomes considerable in the cases where $\Omega_1 \geq 0.15$. Moreover, it follows from the analyses that as a result of the fluid incompressibility the absolute values of the stress T_{22} and the velocity v_2 increase for the considered pair of the fluid and plate materials. However, the character of the fluid compressibility on the velocity v_1 depends on the values of the frequency ω .

Consider the distribution of the studied quantities with respect to y_1/h for the “thick plate case”. The graphs of this distribution are given in Fig. 8a (for T_{22}), Fig. 8b (for v_2) and Fig. 8c (for v_1). The results are obtained for the various values of the plate thickness under $0.05\text{ m} \leq h \leq 1\text{ m}$ in the case where $\lambda_1 = 1.0$. The decay of the values of the T_{22} , v_2 and v_1 with y_1/h is observed clearly from the graphs and this decay becomes more significantly with the plate thickness.

As an example for results related to the phase shifting of the studied quantities we consider the dependence among T_{22} , v_2 and ωt . The graphs of this dependence which are obtained for the various values of the plate thickness under $\lambda_1 = 1.0$ are given in Fig. 9a (for T_{22}) and Fig. 9b (for v_2). It follows from Fig. 9a that the phase shifting for the stress T_{22} is absent in the “thick plate case”. However, this shifting takes place for the velocity v_2 in the “thick plate case” and decreases with the plate thickness.

Now we consider numerical results related to the influence of the initial stretching of the plate on the frequency response of the studied quantities in the “thick plate case”. For this purpose we consider the case where $h = 0.5\text{ m}$ and analyze the graphs given in Fig. 10 a (for T_{22}) and Fig. 10 b (for v_2). It follows from these graphs that, as in the “thin plate case”, the initial stretching of the plate causes

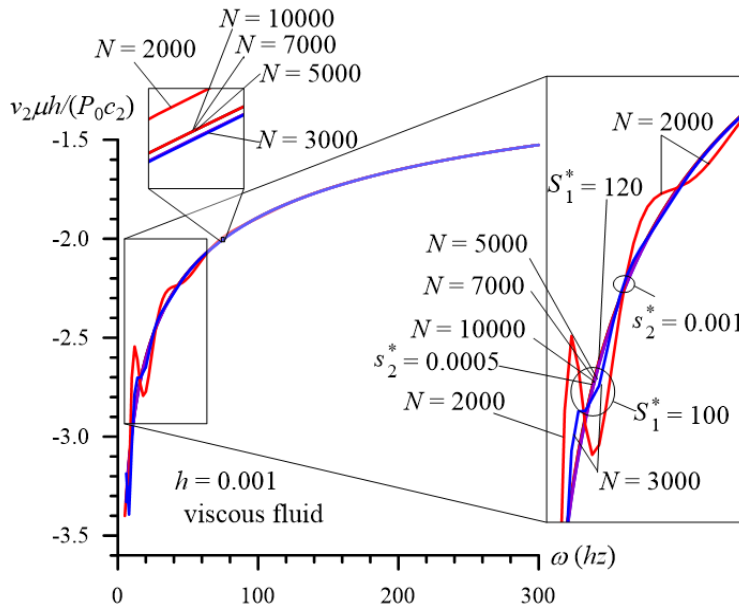


Figure 11: The fragments on the convergence of the numerical results under calculation of the integrals (52).

to decrease in the absolute values of the stress T_{22} and velocity v_2 . Moreover, it follows from the graphs given in Fig. 10 b that, the initial stretching of the plate also changes the character of the frequency response of the velocity v_2 , i.e. in the cases where $1.0 \leq \lambda_1 \leq 1.001$ the absolute values of the v_2 increase, but in the case where $1.005 \leq \lambda_1 \leq 1.20$ decrease monotonically with the frequency ω . The results given in Fig. 10 also show that the T_{22} and v_2 approach to a certain limit values with the parameter λ_1 . For instance, the values of the T_{22} (or v_2) obtained in the case where $\lambda_1 = 1.15$ coincide almost with the corresponding values of that obtained in the case where $\lambda_1 = 1.20$.

Now we analyze an example related to the convergence of the numerical results with respect to S_1^* and s_2^* in the integrals (52). Numerical results obtained for various problem parameters show that the very disadvantaged case in the convergence sense is the “thin plate case” for low frequencies of the external force, namely the case where $h = 0.001$ and $5hz < \omega \leq 300$. As noted above, under calculation of the related integrals, the interval $[0, S_1^*]$ is divided into a certain number of shorter intervals. Let us denote this number through N . Consequently the length of the mentioned shorter intervals is S_1^*/N and in each of these shorter intervals the integration is made by the use of the Gauss integration algorithm with ten sample

point. Consequently, convergence of the mentioned numerical integration can be estimated with respect to the values of S_1^* and N for each fixed value of s_2^* . All the numerical results related to the “thin plate case” have been obtained in the case where $N = 10000$, $S_1^* = 100$ and $s_2^* = 0.001$. Some fragments on the convergence of the numerical results obtained for the velocity v_2 with respect to the N , S_1^* and s_2^* are illustrated in Fig. 11. As follows from Fig. 11, the noted above selected values of the N , S_1^* and s_2^* for calculation of the numerical results are sufficient in the convergence sense. It should be noted that under the “thick plate case” the convergence of the numerical results is achieved in the relatively small values of S_1^* and N .

5 Conclusions

Thus, in the present paper the forced vibration of the system consisting of the pre-strained plate-layer and compressible viscous Newtonian fluid has been studied. The motion of the layer is described within the scope of the three-dimensional linearized theory of elastic waves in initially stressed bodies, but the motion of the fluid within the scope of the linearized Navier-Stokes equations. The elastic relations of the plate material are described with the use of the harmonic potential. It is assumed that the velocities and forces are continuous on the interface plane between the fluid and the plate, and two-dimensional (plane-strain state) problem is considered. Also, it is assumed that the forced vibration is caused by the lineal-located time-harmonic forces acting on the free face plane of the plate. The exponential Fourier transformation with respect to the space coordinate directed along the interface is employed for solution of the corresponding boundary-value and contact problem. The inverse of this transformation is found numerically by employing Sommerfeld contour method. Non-dimensional parameters characterized the compressibility and viscosity of the fluid, are introduced and concrete numerical results related to the interface velocities and normal stress are presented and discussed. These results are obtained in the case where the fluid is Glycerin and the values of the elastic constants which enter in the mentioned above harmonic potential and the density of the plate material are taken as Lamé's constants and density of the Plexiglass (Lucite). Consequently, in the case where the initial strains are absent in the plate material the numerical results relate to the hydro-elastic system consisting of the Plexiglass and Glycerin. According to analyzes of these numerical results, it can be made the following concrete results related to the mechanics of the forced vibration of the system under consideration.

- in the “thin plate case” the influence of the fluid viscosity on its flow velocities is significant and must be taken into account under calculation of these

quantities;

- the magnitude of the aforementioned influence decreases with the frequency of the forced vibration;
- the compressibility of the fluid on the studied quantities is insignificant in the “thin plate case”;
- the initial stretching of the plate causes a significant decrease in the absolute values of the studied quantities;
- the influence of the fluid viscosity on the fluid velocity in the plate laying direction is not only quantitative, but also qualitative, and this conclusion also occurs for the “thick plate case”;
- the influence of the fluid viscosity on the interface pressure and on the fluid flow velocity in the direction which is perpendicular to the interface plane, is insignificant in the “thick plate case”;
- the influence of the compressibility of the fluid on the values of the considered quantities is significant in the “thick plate case”;
- The existence of the fluid constituents in the system under consideration causes to appear the phase shifting of the studied velocities and stress with respect to the phase of the external forces and the magnitude of this phase shifting decrease with the plate thickness and increase with the initial stretching of the plate.

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