

## Researching the Link Between the Geometric and Rènyi Discord for Special Canonical Initial States Based on Neural Network Method

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**Abstract:** Quantum correlation which is different to the entanglement and classical correlation plays important role in quantum information field. In our setup, neural network method is adopted to simulate the link between the Rènyi discord ( $\alpha = 2$ ) and the geometric discord (Bures distance) for special canonical initial states in order to show the consistency of physical results for different quantification methods. Our results are useful for studying the differences and commonalities of different quantizing methods of quantum correlation.

**Keywords:** Neural network method, quantum correlation, rènyi discord, geometric discord.

### 1 Introduction

As part of both artificial intelligence and statistics, machine learning come from the computer science field in which the goal is to learn the potential patterns from prior given data sets. It can make a decision or prediction for future unknown situation based on this learned patterns. Recently, some quantum problems has been study using machine learning method, such as quantum state tomography [Giacomo, Guglielmo, Juan et al. (2018)], and quantum many-body problem [Giuseppe and Matthias (2017)]. The results of these works suggest that machine learning can be a new platform for solving some problems of quantum physics.

Quantum correlation which plays important role in quantum information field is firstly quantified by the concept of "quantum discord" which is introduced by Harold et al. [Harold and Wojciech (2001)] and Henderson et al. [Henderson and Vedral (2001)] in about ten years ago. It show us that there is an universal consensus that entanglement entirely captures

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quantum correlation only for a global pure state [Marco, Thomas, Rosario et al. (2015)], namely the entanglement does not account for all nonclassical correlations and that even the states with zero entanglement usually contain quantum correlations [Harold and Wojciech (2001); Kavan, Aharon, Hu et al. (2012)]. So, many related works have been presented [Xu, Xu, Li et al. (2010); Claudia, Fabrizio, Paolo et al. (2013); Zhu, Ding, Wu et al. (2016); Zhu, Ding, Wu et al. (2015); Zhu, Fu and Lai (2013); Zhu, Ding, Wu et al. (2015); Huang (2014); Li, Zhu, Zhu et al. (2018); Davide, Alexandre, Vittorio et al. (2014); Manabendra (2014); Benjamin, Rosario, Giuseppe et al. (2013); Qu, Zhu, Wang et al. (2018); Liu, Chen, Liu et al. (2018); Qu, Wu, Wang et al. (2017); Qu, Cheng, Liu et al. (2018)]. In general, these works are classified two class for the different quantification methods. One is the entropy style, such as, quantum discord and the Rènyi entropy discord (RED) [Mario, Kaushik and Mark (2015); Kaushik, Mario and Mark (2015)]. The other is geometric quantization methods, such as Hilber-Schmidt ( $D_{HS}$ ), Bures distance [Marco, Thomas, Wojciech, Rosario et al. (2015); Davide, Alexandre, Vittorio et al. (2014); Manabendra (2014)] ( $D_{Br}$ ), trace-norm and Hellinger [Marco, Thomas, Wojciech et al. (2015); Benjamin, Rosario, Giuseppe et al. (2013)] ( $D_{HL}$ ).

From the point of view of invariance of physical laws, even for different methods, the same physical problem should have the same result. So, finding the relation between different quantification methods will help us better study the properties of quantum correlation. Unfortunately, it need face the complex nonlinear mathematical forms of different quantification methods when we wish to resolve this problem from physical view. Looking at this problem from data processing perspective, this problem can be solved by machine learning method. In this work, we extent our works [Zhu, Li, Zhu et al. (2018); Ding, Zhu, Wu et al. (2017)] and further construct the link between  $D_{Br}$  and the RED of  $\alpha = 2$  by the use of machine learning method for special canonical initial states (SCI).

## **2 The link between the $D_{Br}$ and RED( $\alpha = 2$ ) for SCI**

### **2.1 The brief of $D_{Br}$ and RED**

Cianciaruso et al. discussed the geometric measure of dicord-type correlations based on the Bures distance ( $d_{Bu}$ ) [Marco, Thomas, Rosario et al. (2015)], which is defined as follows:

$$D_{Br} \equiv \inf_{\chi'} d_{Bu}^2(\rho, \chi') = \inf_{\chi'} 2(1 - Tr([\sqrt{\chi'}\rho\sqrt{\chi'}]^{1/2})) \quad (1)$$

where the set of classical-quantum states  $\chi' = \sum_i p_i |i\rangle\langle i|^A \otimes \omega_i^B$ ,  $p_i$  is a probability distribution,  $\{|i\rangle^A\}$  denotes an orthogonal basis for subsystem A,  $\omega_i^B$  is an arbitrary ensemble of states for subsystem B, and  $d_{Bu}(\rho, \chi')$  is the Bures distance.

Because it is difficult to obtain mathematically analytic form of Eq. (6) for general models, some numerical calculation methods were proposed in Davide et al. [Davide, Alexandre, Vittorio et al. (2014); Manabendra (2014)] which are also adopted in this work to study  $D_{Br}$  based on the relation between quantum Fisher information and the Bures distance.

The Bures distance can be rewritten

$$\mathcal{P}^A(\rho_{AB}|\Gamma) = \frac{1}{4} \min_{H_A^\Gamma} F(\rho_{AB}; H_A^\Gamma) \tag{2}$$

where  $F$  denotes the quantum Fisher information,

$$F(\rho_{AB}; H_A^\Gamma) = 4 \sum_{i < k: q_i + q_k \neq 0} \frac{(q_i + q_k)^2}{q_i + q_k} |\langle \psi_i | (H_A^\Gamma \otimes I_B) | \psi_k \rangle|^2, \text{ with } q_i, |\psi_i\rangle \text{ denoting respectively the eigenvalues and eigenvectors of } \rho_{AB}, \text{ and the minimum is taken over the set of all local Hamiltonians } H_A^\Gamma.$$

The Rènyi quantum discord of  $\rho_{AB}$  is an extension of quantum discord and is defined for  $\alpha \in (0, 1) \cup (1, 2]$  as follows [Mario, Kaushik and Mark (2015); Kaushik, Mario and Mark (2015)]:

$$D_\alpha(\rho_{AB}) = \inf_{\Pi_k^A} I_\alpha(E; B | X)_{\tau_{XEB}} \tag{3}$$

where the Rènyi conditional mutual information  $I_\alpha(E; B | X)_{\tau_{XEB}}$  satisfies:

$$I_\alpha(E; B | X)_{\tau_{XBE}} = \frac{\alpha}{\alpha - 1} \log Tr \{ (\rho_X^{\frac{\alpha-1}{2}} Tr_E \{ \rho_{EX}^{\frac{1-\alpha}{2}} \rho_{EBX}^\alpha \rho_{EX}^{\frac{1-\alpha}{2}} \} \rho_X^{\frac{\alpha-1}{2}} )^\frac{1}{\alpha} \} \tag{4}$$

where the the classical output  $X$  denotes the measurement acting on system  $A$  and  $E$  is an environment for the measurement map [Kaushik, Mario and Mark (2015)]. In this paper, we choose the von Neumann measurement  $\Pi_{i'} = |i'\rangle\langle i'| (i = 0, 1)$  with two angular parameters  $\theta$  and  $\phi$ :  $|0'\rangle = \cos(\theta/2)|0\rangle + e^{i\phi} \sin(\theta/2)|1\rangle$  and  $|1'\rangle = \sin(\theta/2)|0\rangle - e^{i\phi} \cos(\theta/2)|1\rangle$  ( $0 \leq \theta \leq \pi/2; 0 \leq \phi \leq \pi$ ). The properties of the Rènyi quantum discord are shown in Kaushik et al. [Kaushik, Mario and Mark (2015)].

### 2.2 The sample of SCI states

For the class of canonical initial (CI)states [Mazzola, Piilo and Maniscalco (2010)] of the density matrix

$$\rho_s(0) = \frac{1}{4} \begin{bmatrix} 1 + C_{33} & C_{01} & C_{10} & C_{11} - C_{22} \\ C_{01}^* & 1 - C_{33} & C_{11} + C_{22} & C_{10} \\ C_{10}^* & C_{11} + C_{22} & 1 - C_{33} & C_{01} \\ C_{11} - C_{22} & C_{10}^* & C_{01}^* & 1 + C_{33} \end{bmatrix} \tag{5}$$

The SCI states needs to be satisfied further [Titas, Amit, Anindya et al. (2015)]:

$$\left\{ \begin{array}{l} C_{22}/C_{33} = -C_{11} \\ C_{10}/C_{01} = C_{11} \\ (C_{33})^2 + (C_{01})^2 \leq 1 \end{array} \right. \tag{6}$$

In order to generate more SCI states, we consider the bit-flip (BF) noise channel [Titus, Amit, Anindya et al. (2015)]. In this scenario, it is easy to show that during the evolution of  $\rho(t)$ ,  $c_{11}$ ,  $c_{10}$ , and  $c_{01}$  remain unchanged, whereas the correlations  $c_{aa}$  and the magnetization  $c_{0a}$  and  $c_{a0}$  ( $a = 2, 3$ ) decay with  $\gamma$  as  $(1 - \gamma)^2$  and  $(1 - \gamma)$ , respectively. Here,  $\gamma = 1 - \exp(-\Gamma)$  and  $\Gamma > 0$ . Simultaneously, it is also easy to check the  $\rho_s(t)$  satisfy Eq. (2). Finally, for every sample of SCI states, the values of  $D_{Br}$  and RED can be obtained by the define of  $D_{Br}$  and RED.

### 3 Neural network model

Considering the complex nonlinear mathematical forms of  $D_{Br}$  and  $RED$ , the link between them maybe nonlinear function. So we need to find a method which simulates this nonlinear function. Inspired by the biological neural network we build artificial neural network to study the data. Artificial neural networks is a multi-layer perception model, for each layer we have input data and output data which is the next layer's input. The neural network is used to construct the link between  $D_{Br}$  and RED because the multi-layer neural network can represent any function no matter how complex it is. We train the neural network to adjusted the the weight and bias parameters and use the well-trained neural network to predict the output according to the input we give [Simon (2008)]. Fig. 1 shows a structure graph of our neural network. It has input data  $x_i$ , hidden layer neural  $a_i^l$  and output data  $y$ , satisfying

$$\mathbf{z}^l = W^{(l)}\mathbf{A}^{l-1} + \mathbf{b}_{l-1}; a_i^l = f(z_i^l)$$

where the  $l$ th layer neural cells denote  $\mathbf{A}^l = [a_1^l, a_2^l, \dots, a_n^l]$  and for  $l = 1$ ,  $\mathbf{A}^1 = [x_1^l, x_2^l, \dots, x_n^l]$ ,  $\mathbf{z}^l = [z_1^l, z_2^l, \dots, z_n^l]$ .

The activation function  $f(z)$  is used to realize the nonlinear relation between input and output of each neural node,

$$f(z) = \tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}} \quad (7)$$

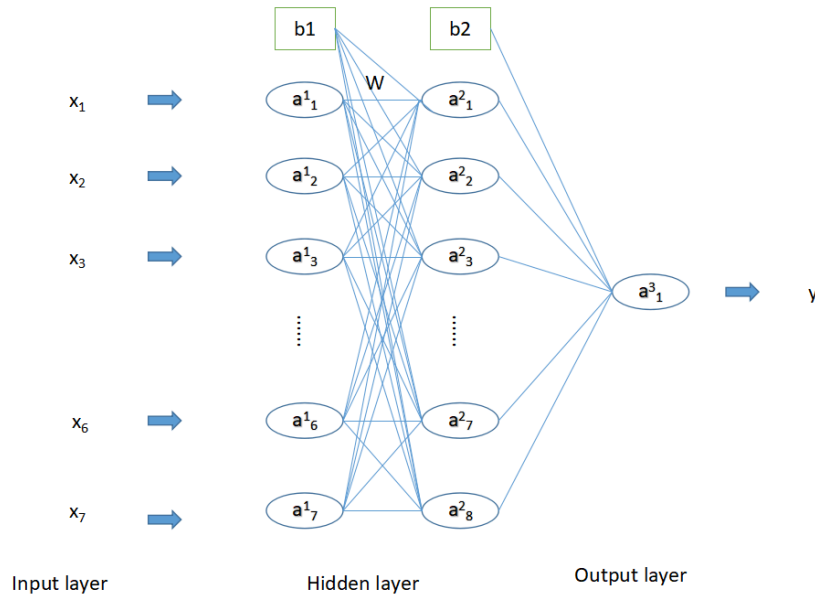
We adjust the parameters of neural network to minimize the cost function by using back-propagation algorithm and gradient descent method.

$$\text{cost}(x) = \sum (y - y')^2 \quad (8)$$

The summation for all the training data(training samples).  $y'$  and  $y$  denote the predicted value and real value of  $D_{Br}$ .

For our problem, the 4 layer neural network is constructed. The number of neurons per layer is 7, 8, 1, 1. The Tab. 1 shows the learning process of neural network.

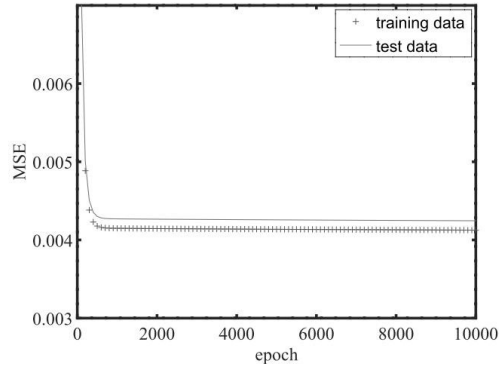
In this paper, we generate 200193 samples under the bit-flip (BF) noise channel (Here,  $C_{11} \in [-1, 1]$ ,  $C_{10}, C_{01}, C_{22}$  and  $C_{33} \in [-0.3, 0.3]$ ,  $t \in [0, 2]$ , with step 0.1.  $\Gamma = [1, 2, 3, 4, 5]$ ). The total number of samples are more than one hundred and twenty thousand with the repetition rate less than 1%. Considering the calculation process of entropy,



**Figure 1:** The structure graph of our neural network.  $b_l$  is the bias unit of  $l$ th layer,  $a_i^{(l)}$  is the  $i$ th neural node of  $l$  layer and the input layer satisfies  $l = 1$ .  $W^{(l)}$  is the weight matrix, the elements  $w_{ij}^{(l)}$  is the weight of connection between  $a_j^{(l-1)}$  and  $a_i^{(l)}$

**Table 1:** Learning process of neural network

Learning process
Input: matrix $n * 7$ ( $n$ data samples with 7 features)
Output: matrix $n * 1$ , the predicted value $y'$ of $D_{Br}$ for each sample
1.Initial the parameters ( $\mathbf{W}, \mathbf{b}$ ) in neural network.
2.Split the data set into training data and test data randomly with proportion 90% and 10%.
for t=1:100000 do
Minimize the difference between predicted value $y'$ and real value $y$ by updating the parameters using gradient descent on training data
End for
3.use the validation data to choose the neural network with the minimal cost function.



**Figure 2:** The figure of the mean-square error (MSE) which is equal to the expectations of *cost* change with epoch. The solid(+) line shows the behavior of test(train) data

the important characteristic parameters of the density matrix are the eigenvalues and the optimized selection of measurement in RED calculation, seven parameters, including the four eigenvalues of  $\rho_s(t)$  and  $\theta$  and  $\phi$  which are introduced in the RED calculation process, and the value of RED, are chosen as the input features of neural network. In Fig. 2, the + line shows that at the end of training, the mean-square error (MSE) which is equal to the expectations of *cost* rapidly decreases at first hundreds epochs and eventually converges after hundred thousand epochs. The MSE is less than 0.0042. This means that a good link is constructed based on our model.

### 3.1 Overfitting

As we apply the machine learning methods, the overfitting need to be carefully avoided in the training process. A regularization technique named dropout is applied to reduce overfitting in neural networks by preventing complex co-adaptations on training data. Dropout can effectively prevent overfitting, which means that we temporarily and randomly remove some units from the network, along with all its incoming and outgoing connections [Nitish, Geoffrey, Alex et al. (2014)]. In Fig. 2, it is shown that after 10000 epoch of training the MSE of training data has reduced to 0.0042 and the MSE of test data is close to that (the magnitudes of the distance between two lines is  $10^{-4}$ ). We can claim that the neural network has constructed without overfitting. It also further demonstrates that we obtain the link between  $RED(\alpha = 2)$  and  $D_{B_r}$  for SCI states. Our results pave a way for the further study of the physical nature of quantum correlation.

Finally, as a conclusion of this section, our neural network model is appropriate and successfully simulated the relationship between geometry and entropy style discord for SCI.

#### **4 Conclusion**

In this paper, we calculate the values of  $D_{Br}$  and RED which are used to quantify the quantum correlation for SCI, and the link between geometric ( $D_{Br}$ ) and entropy ( $RED$ ) style discord is successfully constructed by our neural network model for *SCI*. From the physical perspective, the quantum correlation shows the different characteristics of the quantum states contrasting with the classical states or the changing degree of the quantum states when it suffers the local disturb. So, the system information presented by different discord like definitions will be different. Searching the link between these defines, it will not only help us to understand the differences and commonalities of systematic information obtained by different definitions, but also help us to understand the total properties of quantum states, such as coherence, and the properties of entanglement.

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